Math 171.02 Spring 2004
February 23, 2004
Solutions to Practice Problems

Caveat: errors are possible.

1. Done in class

2. Let S be the event Anita gets an A in statistics, and let B be the event that she gets an A in biology. Then, $P(S) = 0.40$, $P(B) = 0.60$, and $P(S \cup B) = 0.86$.

(a) The event that she does not receive an A in either statistics or biology is $(S \cup B)^c$. Thus, $P((S \cup B)^c) = 1 - P(S \cup B) = 1 - 0.86 = 0.14$.

(b) From the addition law for probabilities we have $P(S \cap B) = P(S) + P(B) - P(S \cup B) = 0.40 + 0.60 - 0.86 = 0.14$. Thus, the probability that she receives A's in both statistics and biology is 0.14 (as well).

3. We must solve the equation

$$\frac{1}{2} = P(\text{red first} \cap \text{red second}) = P(\text{red first}) \cdot P(\text{red second}|\text{red first}) = \frac{3}{n} \cdot \frac{2}{n-1}$$

which gives $n = 4$.

4. Let A be the event that the patient survives the operating table. We are told $P(A) = 1 - 0.20 = 0.80$. Let B be the event that the patient survives the aftereffects. We are told that $P(B|A) = 1 - 0.15 = 0.85$. Hence, $P(A \cap B) = P(B|A) \cdot P(A) = 0.85 \times 0.80 = 0.68$.

5. **(a)** $P(X = 0) = 121/144$, $P(X = 1) = 22/144$, $P(X = 2) = 1/144$ and $P(Y = 0) = 1199/1428$, $P(Y = 1) = 220/1428$, $P(Y = 2) = 9/1428$.

(b) $E(X) = 24/144$ and $E(Y) = 238/1428$

(c) $\mathrm{Var}(X) = 22/144$ and $\mathrm{Var}(Y) = 649/4284$

6. Note that $P(\text{at least 6 correct}) = P(\text{exactly 6}) + P(\text{exactly 7}) + P(\text{exactly 8})$ and each of these probabilities can be computed since the experiment of flipping a coin 8 times and counting the number of heads is binomial with parameters $n = 8$, $p = 1/2$. Thus,

$$\begin{aligned}P(\text{exactly } 6) + &P(\text{exactly } 7) + P(\text{exactly } 8)\\ &= \frac{8!}{6!2!}(1/2)^6(1/2)^2 + \frac{8!}{7!1!}(1/2)^7(1/2)^1 + \frac{8!}{8!0!}(1/2)^8(1/2)^0\\ &= (1/2)^8 \cdot (28 + 8 + 1)\\ &= \frac{37}{256}.\end{aligned}$$

Judge for yourselves if this is convincing evidence that he has ESP as claimed.

7. **(a)** $P(f) = 1/5$, $P(k) = 1/4$, $P(f \cup k) = 3/10$. So $P(f \cap k) = 1/5 + 1/4 - 3/10 = 3/20$. Then $P(k|f) = (3/20)/(1/5) = 3/4$.

(b) $P(f'|k) = 1 - P(f|k) = 1 - (3/20)/(1/4) = 1 - 3/5 = 2/5$.

8. **(a)** 15/36 **(b)** 5/11

9. **(a)** 0.15 **(b)** 0.20

10. **(a)** $15/(15 + 35) = 30\%$ **(b)** $\approx .138$

11. No, $P(E)P(F) = 1/36$ does not equal $P(E \cap F) = 2/36$.

12. **(a)** Let X be normal with mean 100 and SD 16. Then, $P(X < 80) = P(Z < (80 - \mu)/\sigma) = P(Z < (80 - 100)/16) = P(Z < -1.25) \simeq 0.1056$, where Z is normal mean 0, SD 1, and the actual probability is from the table for the standard normal distribution.

(b) $P(80 < X < 120) = P((80 - 100)/16 < Z < (120 - 100)/16) = P(-1.25 < Z < 1.25) = P(Z < 1.25) - P(Z < -1.25) = (1 - P(Z > 1.25)) - P(Z < 1.25) = (1 - P(Z < -1.25)) - P(Z < -1.25) = 1 - 2P(Z < -1.25) = 1 - 2(0.1056) \simeq 0.7888$. Note that, from our work in part **(a)**, no table is needed to solve this part.

(c) $P(X > 140) = P(Z > (140 - 100)/16) = P(Z > 2.5) = 1 - P(Z < 2.5) \simeq 1 - 0.9938 = 0.0062$.

(d) If we consider selecting a child with an IQ higher than 80 as a success, this is just a binomial probability with 5 trials, 4 successes, and probability of success $1 - 0.1056 = 0.8943$. So the probability is

$$\binom{5}{4}(0.8943)^4(0.1056) \simeq 0.3377.$$

13. There are many possible solutions here. A correct solution includes the following elements: if the monkeys were truly choosing the balls that random, then the probability that a given monkey selects the red ball is 1/3. Since there are 1000 monkeys, we expect that 1000/3, or roughly 333, of them will correctly select the red ball.

However, there were 435 that selected the red ball. Thus, the question becomes: how likely is observing 435 red balls from a binomial experiment of 1000 trials when the mean is 1000/3?

We can answer that question with the normal approximation. Suppose that X is a normally distributed random variable with mean $\mu = np = 1000/3 \simeq 333.3$ and standard deviation $\sqrt{np(1-p)} = \sqrt{1000 \times 1/3 \times 2/3} \simeq 14.91$. Then, the chance of observing at least 435 monkeys correctly picking the red ball *if* they were truly choosing randomly is

$$P(X \geq 435) = P(Z \geq \frac{435 - 333.3}{14.91}) = P(Z \geq 6.82).$$

Notice that the z-score 6.82 is not even on Table A. This is becuase this probability is too close to zero to be registered. (Recall that 99.7% of normally distributed data lie within 3 standard deviations of the mean. Thus, less than 0.3% or 0.003 lie outside 3 standard deviations of the mean.)

From the TI-83 we can calculate that

$$P(Z \geq 6.82) \simeq 4.58 \times 10^{-12} = 0.00000000000458.$$

Thus, the chance of 435 monkeys randomly picking the red ball is extremely rare. Thus, there is sufficient evidence *based only on this data* that Prof. Frink's monkeys are intelligent. Hear Prof. Frink himself at:

`http://www.math.cornell.edu/~kozdron/Teaching/Cornell/171Spring04/frink.wav`

14. **(a)** For the secret agent to live until Wednesday, he needs to live to Friday, then Saturday, then Sunday, then Monday, then Tuesday, then Wednesday. The probability of each of these happening is 0.49. Since we want the probability of all of them happening (i.e., their intersection), the answer is $(0.49)^6 \simeq 0.014$.

(b) The probability of any one of the secret agents being alive on Saturday is $(0.49)^2 \simeq 0.24$. This is a binomial distribution, so the expected number of successes (the secret agent living is considered a success) is just the probability of success times the number of trials (i.e., number of secret agents). So the answer is $(0.24)(12) = 2.88$.

15. **(a)** Write the values in order: 150, 180, 190, 230, 250, 250, 280, 300, 340, 380. The median is just the mean of the two middle numbers. Since these two numbers are both 250, the median is 250. The mean is a simple calculation: $(150+180+190+230+250+250+280+300+340+380)/10 = 255$. The standard deviation is calculated just as easily:

$$\sqrt{\frac{697300 - 10(255^2)}{9}} \simeq 72.$$

(b) The Bright Idea Lighting Company's bulbs have a higher mean life.

(c) For the Bright Idea Lighting Company, we have $P'(x > 350) = P(z > (350-262)/41) = P(z > 2.15) = 1 - P(z < 2.15) = 1 - 0.9842 = 0.0158$. For The Electric Company,

$$P'(z > 350) = P(z > \frac{350-255}{72}) = P(z > 1.32) = 1 - P(z < 1.32) = 1 - 0.9066 = 0.0934.$$

16. $P(-z^* < z < z^*) = 1 - 2P(z < -z^*)$. This can be seen clearly from a picture of the standard normal distribution. Now $P(-z^* < z < z^*) = .95 \Rightarrow 1 - 2P(z < -z^*) = .95 \Rightarrow 2P(z < -z^*) = .05 \Rightarrow P(z < -z^*) = .025$. Looking at a table, we see that $-z^*$ is -2.81, and so $z^* = 2.81$.

17. **(a)** Each family is an independent trial. We want the expected number of the 72,069 families that have 3 girls. Thus, a success is "having 3 girls." If the probability of having 3 girls in one 6-child family is q, then the expected number of families with 3 girls is $nq = 72,069q$. We must now compute q.

In each family, q is the probability of having exactly 3 girls. Thus, having 3 successes in 6 trials is a binomial $n = 6$, $p = 1/2$ experiment. This is $q = \frac{6!}{3!3!}(1/2)^3(1/2)^3 = 5/16$.

Finally, the expected number of 6-child families with 3 girls is $72,069 \times 5/16 = 22,521.5625 \approx 22,522$.

(b) The sampling distribution for X is simply the observed frequencies of girls.

$P(X=0) = 1,096/72,069$	$= 0.015$
$P(X=1) = 6,233/72,069$	$= 0.087$
$P(X=2) = 15,700/72,069$	$= 0.218$
$P(X=3) = 22,221/72,069$	$= 0.308$
$P(X=4) = 17,332/72,069$	$= 0.240$
$P(X=5) = 7,908/72,069$	$= 0.110$
$P(X=6) = 1,579/72,069$	$= 0.022$

(c) $E(X) = 0 \cdot P(X = 0) + 1 \cdot P(X = 1) + 2 \cdot P(X = 2) + 3 \cdot P(X = 3) + 4 \cdot P(X = 4) + 5 \cdot P(X = 5) + 6 \cdot P(X = 6) = 3.09$.

(d) This question is poorly worded. Basically, the question asks you to compare how close the observed expected number of girls in a 6-child family (namely 3.09) is to the theoretical binomial$(n = 6, p = 1/2)$ value (namely 3).

Equivalently, the binomial model predicts that there should be $22,522$ 6-child families with 3 girls, while the observed value was $22,221$.

It is up to you to decide if these are "close enough."

(e) Since $X+Y = 6$ always (there are 6 children in each family), we have $E(Y) = 6-E(X) = 2.91$.