Math 171.02 Spring 2004
February 16, 2004
Practice Problems

1. Law of Rare Events

Suppose that you play a game at which you have only a 5% chance of winning. Assuming that each play of the game is independent, what is the probability that you win at least once if you play this game twenty times? What if you only have a 1% chance of winning each play? How does this compare to the *expected* number of wins?

2. Page 182 #16

Anita has a 40% chance of receiving an A grade in statistics, a 60% chance of receiving an A grade in biology, and an 86% chance of receiving an A grade in either statistics or biology. Find the probability that (a) she does not receive an A in either statistics or biology; (b) she receives A's in both statistics and biology.

3. Page 204 #16

There are n socks in a drawer, of which 3 are red. Suppose that if 2 socks are randomly chosen, then the probability that they are both red is 1/2. Find n.

4. Page 204 #21

At a certain hospital, the probability that a patient dies on the operating table during open-heart surgery is 0.20. A patient who survives the operating table has a 15% chance of dying in the hospital from the aftereffects of the operation. What fraction of open-heart surgery patients survive both the operation and its aftereffects?

5. Page 229#9

A shipment of parts contains 120 items of which 10 are defective. Two of these items are randomly chosen and inspected. Let X denote the number that are defective (assuming replacement), and let Y denote the number that are defective (assuming no replacement). (a) Find the probability distributions of X and Y. (b) Find $E(X)$ and $E(Y)$. (c) Find $\text{Var}(X)$ and $\text{Var}(Y)$.

6. Page 258 #11

A man claims to have extrasensory perception (ESP). As a test, a fair coin is flipped 8 times, and he is asked to predict the outcomes in advance. Suppose that he gets 6 correct answers. What is the probability that he would have gotten at least this number of correct answers if he had no ESP but had just guessed?

7. One out of five students eating at Trillium order french fries, and one out of four get ketchup. Three out of ten get either french fries or ketchup.

(a) What is the probability that a student who orders fries gets ketchup?

(b) What is the probability that a student who gets ketchup does not order fries?

8. Two fair dice are rolled.

(a) Find the probability that the sum is no more than 6.

(b) Find the probability that the sum is at least 9 given that at least one of the die shows a 5.

9. To survive the pressures of university, many students resort to consuming caffeine. Common sources of caffeine are coffee, tea, and cola.

A survey was conducted among Cornell students and that it was found that 55% of them drink cola, 45% of them drink coffee, and 25% of them drink tea.

It was also found that 15% drink both tea and cola, 5% drink all three beverages, 25% drink both cola and coffee, and 5% drink only tea.

Suppose that a Cornell student is selected at random.

(a) What is the probability that the student drinks only coffee?

(b) What is the probability that the student drinks none of these beverages?

10. According to Dr. Jim Mass in Psychology 101, 43% of adults are moderately to severely sleep deprived, and "college students are walking zombies." It is has been found that college students require at least 9 hours sleep, and that caffeine causes fragmented sleep.

In the same survey of Cornell students as in the previous problem, it was found that 50% of them sleep for less than 6 hours each night, 35% of them sleep between 7 and 9 hours each night, and only 15% of them sleep for over 9 hours each night.

(a) Suppose that a Cornell student is chosen at random, and that the student sleeps for more than 7 hours each night. What is the probability that the student actually sleeps for more than 9 hours each night?

(b) Suppose that 5 Cornell students are chosen at random. What is the probability that exactly 2 of them receive the required sleep, namely over 9 hours each night?

11. Suppose you roll a standard six-sided die twice. Let E be the event, "the first roll is a 5," and let F be the event, "the sum of the two rolls is greater than 9." Are E and F independent? Why or why not?

12. The intelligence quotient (IQ) score, as measured by the Stanford–Binet IQ test, is normally distributed in a certain population of children. The mean IQ score is 100, and the standard deviation is 16 points. What percentage of children in the population have IQ scores

(a) of 80 or less?

(b) between 80 and 120?

(c) of 140 or more?

(d) Suppose that five children are chosen at random from the population. What is the probability that one of them will have an IQ score of 80 or less and four will have IQ scores higher than 80?

13. **(Open-Ended)** In a famous experiment designed to test monkey intelligence, Prof. John Frink performed the following experiment on 1000 monkeys. He gave each monkey a red ball, and allowed the monkey to play with it for one minute. He then placed the same red ball along with a blue ball and a yellow ball into a box, and instructed the monkey to select the red ball. Prof. Frink recorded the ball chosen by the monkey. His data are summarized below:

ball selected	# of monkies
red	435
blue	258
yellow	311

Based on the results of his experiment, do you believe that there is adequate evidence for Prof. Frink to conclude that the monkies are able to correctly distinguish the red ball, and therefore exhibit intelligence?

14. In the theme song to the 1960's television show "Secret Agent Man", we're told, "Odds are he won't live to see tomorrow." Suppose that on any given day that he's alive, the probability that the secret agent lives to see tomorrow are 49%.

(a) Today is Thursday. What's the probability that he'll still be alive next Wednesday?

(b) Suppose that there is a team of 12 secret agents. If each lives or dies independently of the others, how many do you expect to be around on Saturday?

15. Bright Idea Lighting tests their light bulbs, and finds that they have a mean life of 262 hours, with a standard deviation of 41 hours. They test a sample of light bulbs of their rival, The Electric Company, and get that they last 340, 190, 150, 280, 250, 180, 380, 300, 250, and 230 hours.

(a) Find the median, mean, and standard deviation of the life of The Electric Company's light bulbs.

(b) Which brands' light bulbs have the higher mean life?

(c) Assuming the distribution of bulb life of both companies follows a normal distribution, how likely is each company to produce a light bulb that lasts 350 hours?

16. Suppose that a certain population of observations is normally distributed. Find the value of z^* such that 95.00% of the observations in the population are between $-z^*$ and z^* on the Z-scale.

17. In a classic study of the human sex ratio, families were categorized according to the sexes of the children. Data were collected in Germany in the 19th century, when large families were common, and revealed that there were 72,069 families with six children. Prior to this survey, it was widely believed that the distribution of the number of girls in a family with six children was binomially distributed with $n = 6$, $p = 1/2$.

(a) Assuming that the distribution of the number of girls in a family with six children is binomial with $n = 6$, $p = 1/2$, how many of these 72,069 families are expected to have 3 girls?

The following table shows the results for 72,069 families with 6 children.

# of boys	# of girls	# of families
0	6	1,096
1	5	6,233
2	4	15,700
3	3	22,221
4	2	17,332
5	1	7,908
6	0	1,579

Suppose that X is the number of girls in a family with six children chosen at random, and suppose that Y is the number of boys in a family with six children chosen at random.

(b) Determine the sampling distribution for X and plot the corresponding histogram.

(c) Compute $E(X)$, the mean number of girls in a family with six children.

(d) How does your answer in **(c)** compare with your answer in **(a)**? Is this sufficient evidence to conclude that the distribution of the number of girls in a family with six children is binomial with $n = 6$, $p = 1/2$.

(e) Compute $E(Y)$, the mean number of boys in a family with six children. *(Hint: How are X and Y related?)*