Solutions of homework 9

14.2 (a) If μ = 115, the distribution is approximately Normal with mean μ = 115 and standard deviation $\sigma/\sqrt{25}$ = 6.

(b) The actual result lies out toward the high tail of the curve, while 118.6 is fairly close to the middle. If μ = 115, observing a value similar to 118.6 would not be too surprising, but 125.7 is less likely, and it therefore provides some evidence that μ > 115.

14.6 H_0: μ = 5 mm; Ha: $\mu \neq$ 5 mm. μ is the mean diameter of all spindles, while $\bar{x}$ is the mean diameter of only those spindles in our sample.

14.22 (a) z ≈ -2.20.

(b) This result is significant at the 5% level because z < -1.960.

(c) It is not significant at 1% because z >= -2.576.

(d) The absolute value of this value of z is between 2.054 and 2.236, so the P-value is between 0.02 and 0.04 (because the alternative is two-sided).

14.24 (a) Yes: P = 0.06 indicates that the result observed are not significant at the 5% level, so the 95% confidence level will include 10.

(b) No: Because P < 0.1, we can reject H_0: μ = 10 at the 10% level. The 90% confidence interval would include only those values k for which we could not reject H_0: μ = k at the 10% level.

14.27 Our hypotheses are H_0: μ = 100; Ha: $\mu \neq$ 100. We have known that $\bar{x}$ = 105.84, so the test statistic is z ≈ 2.17, and the P-value is P = 2P(Z > 2.17) = 0.0300. This is strong evidence (significant at the 5% level) that the mean IQ differs from (is greater than) 100.

14.28 Our hypotheses are H_0: μ = 25μ g/l; Ha: μ > 25μ g/l. We have known that $\bar{x}$ = 30.4μ g/l, so the test statistic is z ≈ 2.44, and the P-value is P = P (Z > 2.44) = 0.0073. This is strong evidence against H_0; we conclude that the student's mean threshold is greater than 25μ g/l.

15.5 (a) z = 1.64 < 1.645 -- not significant at 5% level (P = 0.0505).

(b) z = 1.65 > 1.645 -- significant at 5% level (P = 0.0495).

15.14 (a) Reject H_0 if z < -2.236.

(b) The probability of making a Type I error is 0.01 (α, the significant level).

(c) We accept H_0 if z >= -2.236, which corresponds to $\bar{x}$ >= 270.185. Then P (Type II error when μ = 270) = P ($\bar{x}$ >= 270.185 given μ = 270) ≈ P (Z >= 0.09) = 0.4641.

15.31 (a) $|z|$ >= 2.576 is equivalent to z <= -2.576 or z >= 2.576, so we reject H_0 if $\bar{x}$ <= 0.84989 or $\bar{x}$ >= 0.87011. In other words, we reject H_0 if $\bar{x}$ is not between 0.84989 and 0.87011.

(b) The power against μ = 0.845 is approximately 1 – P (1.25 < Z < 6.40) ≈ 0.8944.

(c) P (Type II error) = 1 – Power = 0.1056.