Solution of homework 4

3.24

(a) About 5%: $x < 240$ corresponds to $z < (240-266)/16 = -1.625$. Table A gives 5.16% for – 1.63 and 5.26% for – 1.62.

(b) About 55%: $240 < x < 270$ corresponds to $(240-266)/16 < z < (270-266)/16$, which is $-1.625 < z < 0.25$. The area to the left of 0.25 is approximately 60%; subtracting the area to the left of – 1.625 (which is the answer 5% from part a) leaves about 55%.

(c) About 279 days or longer: Searching Table A for 0.80 (which is 1- 20%) leads to $z > 0.84$, which corresponds to $x > 266 + 16(0.84) = 279.44$.

3. 28

(a) The area to the left of Q(1) should be 0.25, which places it about – 0.67. Similarly, Q(3) is about 0.67.

(b) For any normal distribution, the quartiles are 0.67 and – 0.67 standard deviations for the mean; for human pregnancies, the quartiles are about 266+10.7 and 266 - 10.7, or 255.3 and 276.7 days.

3.29

(a) Search Table A for 0.10 and 0.90; the deciles are about 1.28 and – 1.28.

(b) 64 -1.28*2.7 and 64 + 1.28*2.7, or about 60.5 and 67.5 inches.

12.22

(a) With $n = 100$, the mean and standard deviation are $\mu = 75$ and $\sigma \approx 4.3301$ questions. So $P(X \le 100*70\%) = P(X \le 70) = P(Z \le (70-75)/4.33) = P(Z \le -1.15) = 0.1251$.

(b) With $n = 250$, the mean and standard deviation are $\mu = 187.5$ and $\sigma \approx 6.8465$ questions. So $P(X \le 250*70\%) = P(X \le 175) = P(Z \le (175-187.5)/6.85) = P(Z \le -1.83) = 0.0336$.

12.24

(a) The mean $\mu = 1200*0.0996 = 119.52$ Hispanics.

(b) The standard deviation is $\sigma \approx 10.37$ Hispanics, so $P(X \le 100) = P(X \le 175) = P(Z \le (100-119.52)/10.37) = P(Z \le -1.88) = 0.0301$.

18.1

(a) The population is “all college students”. P is the proportion of the population who say they pray at least once in a while.

(b) $\hat{p} = 107/127 \approx 0.8425$.

18.2

(a) The population is Internet users. p is the proportion of the population who would seek health/medical information on the Internet.

(b) $\hat{p} = 606/1318 \approx 0.4598$

18.3

(a) The mean is p = 0.5 and the standard deviation is $\sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{0.25}{14941}} \approx$ 0.004091.

(b) P(0.49 < $\hat{p}$ < 0.51) ≈ P($\dfrac{0.49-0.5}{0.004091}$ < Z < $\dfrac{0.51-0.5}{0.004091}$) ≈ P(-2.44 < Z < 2.44) = 0.9854.

18.5

For n = 1000: P(0.49 < $\hat{p}$ < 0.51) ≈ P(-0.63 < Z < 0.63) ≈ 0.4714.

For n = 4000: P(0.49 < $\hat{p}$ < 0.51) ≈ P(-1.26 < Z < 1.26) ≈ 0.7924.

For n = 16000: P(0.49 < $\hat{p}$ < 0.51) ≈ P(-2.53 < Z < 2.53) ≈ 0.9886.

Conclusion: Larger sample sizes give more accurate estimates. (When the sample size is large, the probability that the sample proportion mean is really close to the true population mean is also large)