Math 135 (Summer 2006)
Warm-up Exercises for July 12, 2006

1. According to the *National Lightning Safety Institute* the chance of being struck by lightning is roughly 1/280 000. (http://www.lightningsafety.com/nlsi_pls/probability.html)

The card game of bridge is played with a standard deck of 52 cards by 4 players. Each player is dealt 13 cards. A perfect bridge hand occurs when a player is dealt all 13 cards of a single suit. Suppose that the deck is well shuffled, and the top 13 cards are dealt to the first player.

To play the New York "Lotto" you select 6 different numbers from among $\{1, 2, 3, \ldots, 59\}$. Each week 6 numbers are drawn at random without replacement by the Lottery Corporation. In order to win the grand prize a player's 6 numbers must match the 6 numbers drawn that week. (The order the winning numbers are drawn does not matter.)

Pat is a compulsive gambler and plays both bridge and "Lotto" each week. Consider the events (a) as first player, Pat will be dealt a perfect hand at bridge; (b) Pat will win the "Lotto"; and (c) Pat will be struck by lightning.

Which of these three events is most likely to occur this week? Why?

2. A food science class at Cornell makes a new flavor of ice cream by adding extra ingredients to vanilla ice cream. This year, the possible ingredients are: chocolate chips, peanuts, marshmallows, walnuts, cherries, and caramel.

(a) How many flavors can they make using exactly two additional ingredients?

(b) How many flavors are possible using any number of additional ingredients (including plain vanilla ice cream)?

(c) Suppose we make a pint of all of the possibilites in **(b)**. What is the probability that it has no nuts in it?

3. You are handing out Hallowe'en candy to trick-or-treaters; you have a giant bowl filled with 22 Snickers, 35 Kit Kats, and 20 Almond Joy bars. A particularly hungry fairy princess grabs five candy bars at random from your bowl. What is the probability that she took:

(a) 3 Kit-Kats, 1 Almond Joy, and 1 Snickers?

(b) only candy with nuts? (Snickers and Almond Joy have nuts, Kit Kat does not.)

(c) five of the same kind of candy bars?

4.

(a) How many different words can be written using all six letters of "banana"? (Assume that a word is a combination of letters, whether or not that word is in a dictionary.)

(b) How many 3 letter words can be written using the six letters of "banana"? (Again assume that a word is a combination of letters, whether or not that word is in a dictionary.)

Solutions to Selected Problems

3.

(a) There are 35 Kit-Kats of which she chooses 3, and there are 20 Almond Joys of which she chooses 1, and there are 22 Snickers of which she chooses 1. By the multiplication principle, this can be done in

$$\binom{35}{3} \times \binom{20}{1} \times \binom{22}{1}$$

ways. Since there are $\binom{77}{5}$ total ways she can grab 5 candy bars, the probability that she took 3 Kit-Kats, 1 Almond Joy, and 1 Snickers is

$$\boxed{\frac{\binom{35}{3} \times \binom{20}{1} \times \binom{22}{1}}{\binom{77}{5}}.}$$

(b) There are $22 + 20 = 42$ candy bars with nuts. Thus, she can choose 5 bars with nuts in $\binom{42}{5}$ ways so that the probability that she took 5 bars with nuts is

$$\boxed{\frac{\binom{42}{5}}{\binom{77}{5}}.}$$

(c) In order to choose 5 of the same kind of candy bars she could have chosen either 5 Kits-Kats, which could be done in $\binom{35}{5}$ ways, or she could have chosen 5 Almond Joys, which could have been done in $\binom{20}{5}$ ways, or she could have chosen 5 Snickers, which could have been done in $\binom{22}{5}$ ways. Thus, the probability that she took 5 of the same kind of candy bar is

$$\boxed{\frac{\binom{35}{5} + \binom{20}{5} + \binom{22}{5}}{\binom{77}{5}}.}$$

4.

(a) There are 6! ways to line up the six letters 'b', 'a', 'n', 'a', 'n', 'a'. However, there are 3! repetitions due to 'a', and there are 2! repetitions due to 'n'. Thus, there are

$$\boxed{\frac{6!}{3!2!} = 60}$$

different words that can be written.

(b) The only way to solve this problem is to write out the possibilities. Note that there are three 'a's and two 'n's and one 'b'.

Thus, if we want to use three 'a's there is one possibility, namely 'aaa'.

If we want to use two 'a's we can combine them with one 'n' in $3!/2! = 3$ ways, namely 'naa', 'ana', 'aan'.

If we want to use two 'a's we can also combine them with one 'b' in $3!/2! = 3$ ways, namely 'baa', 'aba', 'aab'.

If we want to use two 'n's we can combine them with one 'a' in $3!/2! = 3$ ways, namely 'ann', 'nan', 'nna'.

If we want to use two 'n's we can also combine them with one 'b' in $3!/2! = 3$ ways, namely 'bnn', 'nbn', 'nnb'.

Finally, if we want to use the lone 'b', then we must combine it with one 'a' and one 'n' (since all other possibilities have been accounted for). This can be done in $3! = 6$ ways, namely 'ban', 'bna', 'abn', 'anb', 'nba', 'nab'.

Thus, there are $\boxed{19}$ possible three letter words.