**Math 111 Prelim #1 - July 7, 2003**

**This exam has 7 problems and 2 pages**.

You have 75 minutes to complete this exam. You may use a calculator, but no other aids are allowed. In order to receive full credit, you must justify your solutions. No credit will be awarded for having only the final answer, whether or not it is correct.

**1.** (*20 points*) Compute the following limits.

**(a)** $\lim_{x\to -1} \dfrac{2x^2+x-1}{x+1}$

**(b)** $\lim_{x\to -1} \dfrac{2x^2+x-1}{2x+1}$

**(c)** $\lim_{x\to \infty} [\ln(x^2) - \ln x]$

**(d)** $\lim_{x\to -\infty} \dfrac{3x^3+2x-5x^{-2}}{2x^{-2}-x^2-2x^3}$

**2.** (*20 points*) Suppose that the function $f$ is given by

$$f(x) = \frac{x}{\sqrt{x^2-1}}.$$

**(a)** Find $\mathscr{D}(f)$, the domain of $f$.

**(b)** Find $f^{-1}$, the inverse function of $f$.

**(c)** Find $\mathscr{D}(f^{-1})$, the domain of $f^{-1}$.

**(d)** Find all asymptotes of $f$, both horizontal and vertical.

**3.** (*15 points*) Consider the function $f$ where

$$f(x) = \frac{1}{\sqrt{x+1}}.$$

Using the definition of derivative directly, find the equation of the tangent line to the curve $y = f(x)$ at the point $x = 3$. *(Note that absolutely no credit will be awarded for using any "shortcuts" for differentiation.)*

**4.** (*10 points*) Let the function $f$ be defined by

$$f(x) = \begin{cases} 3x^2\cos^2(1/x), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Is $f$ continuous at $x = 0$? Prove or disprove.

**5.** (*10 points*) Prove that the real number $\sqrt{3}$ exists by carefully showing that there is a number $c$ such that $c^2 = 3$.

**6.** (*15 points*) The distance traveled by a car is given by the values in the following table.

| $t$ (seconds) | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $d$ (feet) | 0 | 10 | 32 | 70 | 119 | 178 |

**(a)** Use the data to sketch the graph of $d$ as a function of $t$. Is it reasonable to assume that $d$ is a continuous function of $t$?

**(b)** Estimate the velocity of the car at $t = 3$ seconds.

**(c)** Using both the graph and your answer in **(b)**, estimate the distance traveled after 3.5 seconds.

**7.** (*10 points*) Suppose that $f$ is an odd function with $f(-2) = -1$. It is also known that

$$\lim_{x \to 2} f(x) = 1.$$

**(a)** Is $f$ necessarily continuous at $x = 2$?

**(b)** Must $f$ have a root in the interval $(-2, 2) = \{x : -2 < x < 2\}$?