What's the Big Idea?

We mathematicians are fond of telling our students that mathematics is not a spectator sport. Watching someone else test infinite series for convergence engenders no facility in testing series oneself. Lately, however, the mainstream press has shown a laudable interest in eavesdropping on developments in mathematical research. I am encouraged to see reporters trying to convey both the excitement of mathematical discovery and a hint of the underlying mathematical ideas.

After the hoopla surrounding Andrew Wiles's proof of Fermat's Last Theorem died down, years passed during which the words "elliptic curve" did not appear in the *New York Times* except in passing (as in the 1998 obituary of André Weil). Then a news item by Sara Robinson in July 2002 not only used the words but also gave some indication of the mathematics underlying the reconstruction by Hendrik W. Lenstra Jr. and Bart de Smit of Escher's lithograph "Print Gallery" (see the cover story in the April 2003 issue of the *Notices*).

In August 2002 the mathematical world was flabbergasted by the new algorithm of Agrawal, Kayal, and Saxena for testing primality of integers (see the article by Folkmar Bornemann in the May 2003 issue). The three researchers posted a preprint on the Internet, and within days Sara Robinson published a story about the breakthrough. Writing for the *New York Times* audience, she could not use mathematical language to describe the new algorithm as proving that the problem of determining primality belongs to the polynomial-time complexity class P, but her article did suggest the frenzy of activity that was occurring as the new idea circulated around the mathematical community.

This year has seen a similar ferment among number theorists about a preprint of Dan Goldston and Cem Yıldırım on "Small gaps between primes". Nobody knows how to attack the twin-prime conjecture, which states that there are infinitely many prime numbers p for which $p + 2$ is also prime, but Goldston and Yıldırım developed a new technique that seemed to smash all previous records about how close together consecutive primes are infinitely often.

I am pleased that the press reported the story coherently. On April 9 a news item in the journal *Nature* stated "Around a number x in the sequence of integers, the average spacing between consecutive primes is proportional to the logarithm of x. … Goldston and Yıldırım have shown that the gaps between consecutive primes can indeed be significantly smaller than the average ($\log x$)."

"The idea is the important thing," said mathematician and songwriter Tom Lehrer in his 1965 performance at the hungry i nightclub. Satirizing the "New Math", he used then-current mathematical pedagogy to obtain a wrong answer to an arithmetic problem, and the audience burst into laughter at his comment. Despite the humorous context of the remark, it has more than a grain of truth. We mathematicians do pride ourselves on our rigorous logic and on our attention to detail, but above all we value the big ideas.

As this issue of the *Notices* goes to the printer, there appears to be a problem in the proof of Goldston and Yıldırım: some error terms originally thought to be of lower order are actually of the same size as the main term. Whatever the ultimate results turn out to be, I hope that future stories in the press describe the work of Goldston and Yıldırım as a success story: Their intriguing original idea has all the experts burning the midnight oil to figure out the implications. How many of us can say the same about our research?

Topologists too were abuzz this spring about a dramatic development in their field: the low-key announcement by Grisha Perelman that he can prove William P. Thurston's geometrization conjecture by adapting Richard Hamilton's program. A corollary would be the Poincaré conjecture, that every simply connected, compact, three-dimensional manifold without boundary is homeomorphic to the three-sphere. Perelman has posted to the arXiv two preprints on his work (November 11, 2002, and March 10, 2003), with more promised.

Although Perelman is not giving interviews ("premature"), the \$1 million prize offered by the Clay Mathematics Institute for a proof of the Poincaré conjecture makes the story irresistible. In the span of six days in April, the *New York Times* ran two articles about Perelman's attack on the Poincaré conjecture. I find it noteworthy that these articles use language that we mathematicians ourselves might employ. Checking Perelman's work will take some time, says one article, but in view of his new ideas, "[I]t is clear that his work will make a substantial contribution to mathematics." Says the other article: "That grown men and women can make a living pondering such matters is a sign that civilization, as fragile as it may sometimes seem, remains intact."

I hope that the popular press will continue to portray mathematics as being like a diamond: extremely hard material, but valuable and highly prized both for its industrial applications and for its intrinsic beauty.

—Harold P. Boas
Editor

Letters to the Editor

Mathematical Word Processing

While I respect the right of William C. Hoffman to his own opinion about Mathematical Word Processing (March 2003 issue), I deeply disagree with him. Writing mathematics involves much more than just writing. You write, you send what you wrote to coauthors, you read what they sent you, you make corrections, etc. While maybe TeX is not ideal for those purposes, it is the best. Every mathematician using a computer for writing papers has his or her favorite text editor. TeX fits to all of them. I agree that Microsoft Word is present on more computers than TeX, but also many more people hate it. It produces giant files that require specialized software to read. When I am receiving an email from my coauthor, in which he quotes several lines from the paper we are writing, I can understand it independently of whether I am using Microsoft Outlook Express in the Windows environment or the Unix command-line mail on a SUN.

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William C. Hoffman ("Letters", March 2003) argues the inferiority of TeX and its derivatives, henceforth called "LaTeX" for brevity: "The AMS has been captured by this ponderous, unintuitive, error-ridden software, and we are all hostage to the obsolescence." His suggested alternative, MathType and Microsoft Word, is undesirable for several reasons (none of which apply to LaTeX), but is also arrived at via a false dichotomy: that authors must choose between machine-friendliness and user-friendliness.

A mark-up language based on open standards, such as LaTeX or MathML, is *the* logical choice for disseminating and archiving academic literature. Mark-up separates a document's abstract structure from its appearance, which in turn facilitates machine processing. This basic capability is essential to the consistency of print journals and to the utility and longevity of electronic documents. (The assertion of some LaTeX proponents that separating form and content encourages logical writing is at best a fringe benefit.) To ensure documents' viability into the unforeseeable future, it is equally crucial that file formats be based on open standards, neither proprietary nor encumbered by patents.

Most word processor file formats are neither human-readable nor based on open standards, as both goals are counter to the aims of commercial software. File formats are deliberately obscure to thwart would-be competitors. Closed file formats all but force users to upgrade when a new version is released, since by design new files cannot be read with older software. This may be good business, but it is bad for free exchange of information.

Files stored in proprietary format are available only to someone who has purchased a specific vendor's product. This puts a substantial and unnecessary financial burden on individuals and organizations, which is exacerbated for users in developing countries. LaTeX is available at low or no cost for all common operating systems and is bundled with the legally sharable operating system GNU/Linux, making LaTeX available to anyone with a computer.

The AMS may have additional reasons for strongly preferring LaTeX files for electronic submission, but I am confident that "lock in" due to "early random events" in the history of mathematical typesetting is not among them.

Hoffman also writes, "Freely writing mathematics has been replaced by programming in a ponderous system of macros." Readers who share Hoffman's frustration might be interested in LyX (www.lyx.org), a free (both GPL and no-cost) word-processor-like interface to LaTeX that runs on multiple platforms.

LaTeX is among the oldest and stablest software applications in wide use. In fifteen years of LaTeX-ing, I have never encountered "macros that become obsolete from one edition to the next," so this charge is impossible to answer. However, given Word's release history and notorious backward compatibility problems, Hoffman's criticisms are particularly ironic.

As for plain TeX, its features may not please everyone, but it does exactly what it claims to and is arguably the most bug-free large program in wide use. No commercial software comes close.

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Mathematics Education

Steven Zucker's article [March 2003 "Opinion"] treats by design only informing students of their responsibilities in our basic courses. While it is important to communicate those things effectively, he doesn't address the cognitive difficulties that students have. These difficulties occur because there are no warnings in going from high school to college, and I wish to bring these up. Some of these can't be addressed unless the instructor handles them directly in class. More than Zucker seems to allow, they should be the responsibility of the instructor during class time. Most of our students cannot grapple with many of the intellectual difficulties related to our approach to mathematics on their own.

Here is a cognitive difference between high school and the university. High school classes study specific functions like sin and polynomials. They don't study collections of functions. Further, high school texts test students on memorable properties of a standard set of functions, often quadratic polynomials or sin. Students don't get much chance to consider the set of functions that satisfy certain properties. It's too time-consuming for class. Yet, it requires instructor guidance for students to catch on. It can't be left for the out-of-class exercises.

A major jump between high school and college is that students in college will never again be tested on properties of functions that they have learned intensely. In the end, they don't know functions at all; they know this function and that function.

That's a difference between high school and college. Here's a similarity.

High school material goes much too fast for almost all students. It just wooshes by. Ditto for college. There's no time to practice independent problem solving. None of my colleagues do problem solving of a slow analytic nature in class. They haven't found a way to weave occasional analysis into a 45-minute presentation on one problem as if it were a serious mathematical story with an interesting punchline. There is no time for students to have the analysis register. The reality for students is that independent problem solving is too slow a process for them to get much from it. What they do won't match the speed of presentation of almost all instructors at research universities. Students feel bad taking forever to accomplish what the instructor does in two minutes.

Students may like you. You may get good evaluations. Still, anyone can see that from one class to the next, few students get much from the material. Upon their failure to handle problem-solving assignments, they will tell you they worked for hours.

You may not believe them. I do. They worked for hours, in the sense of having struggled without a clue for what to focus on. The only remedy I know is to slow down in class. Take some time showing them how to read the material. Find out what their skills are, and then try to build on them using their still-limited vocabulary. This will require some real people intuition and empathy for students. You won't cover the whole syllabus either. Still, by doing this, you will develop students who have seen a higher analytical standard, from responses from some students in the class. You might not get through all the material in those big books we get our syllabi from. Yet slowing down at the beginning sometimes means going at a reasonable rate at the end, where many classes just fall apart.

Oh, yes. That is another difference between high school and college. College students find fewer of their instructors have people intuition and empathy.

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Response to Fried

Mike Fried's point about cognitive differences and similarities is both interesting and enlightening. Of course, we all face similar issues in our courses. A good part of the apparent difference in our positions may be a consequence of the difference in the nature of our student bodies. He and I do seem to agree that one should not waste too much time in class on easy things and should use the time to reinforce the main conceptual points and their role in solving problems.

Many of us have people intuition and empathy; it's just that we may not use them in the way that a high school teacher (with different aspirations) does. Instead, we want the student to accept major responsibility in learning the subject and be willing to use resources outside of the classroom. By the way, the textbook, though sometimes hard to read, goes at the reader's pace.

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About the Cover

Hidden Symmetries

This month's cover arose from a suggestion of Ravi Vakil in order to illustrate a point made in his article on the moduli of algebraic curves (p. 650, top of first column). It portrays the action of a discrete group of non-Euclidean transformations whose quotient is the curve of genus three possessing as automorphism group the projective matrix group $PSL_2(\mathbb{Z}_7)$, the largest possible for a curve of that genus. The large assembly of colored regions makes up a fundamental domain for the fundamental group of this curve, and the picture shows that the curve is associated to a non-Euclidean tiling by heptagons possessing the maximum degree of symmetry. As with all such tilings, the automorphism group of this tiling is a Coxeter group with a nonbranching Coxeter diagram, in this case with links of multiplicities 3 and 7. The small triangles in the figure are the chambers of this Coxeter group. The cyclic symmetry of the curve of order 7 is apparent in the figure, but the other symmetries are more subtle. The curve may be constructed explicitly by gluing together boundary regions according to color matching around the edge.Other regular non-Euclidean tesselations may be seen at http://www.hadron.org/~hatch/HyperbolicTesselations/. The whole book *The Eightfold Way* by Silvio Levy and others (Cambridge University Press, 1999) is devoted to this remarkable figure. It includes a translation by Silvio Levy of the classic article (*Math. Ann.* **14**) by Felix Klein on this figure. The German original is available from the digitization project at http://134.76.163.65:80/agora_docs/29551TABLE_OF_CONTENTS.html. The figure was constructed with the help of a word recognizer for the Coxeter group built by Derek Holt's program kbmag. The apparent asymmetry in the boundary is there because the graphics program I used drew all chambers at a fixed word length from one of them near, but not at, the centre.

I wish to thank Silvio Levy for help in locating Klein's figure.

—Bill Casselman
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