Math 111.01 Summer 2003
July 18, 2003

Prelim #2 Review

These are some of the main ideas in Chapters 2, 3, and 4. You should also review the main ideas in Chapter 1. The prelim will be comprehensive, *with an emphasis on material covered since Prelim #1.*

1. What does it mean for a limit to exist from the right? from the left?
2. State the definition of continuity at a point. In other words, what does it mean for the function $f(x)$ to be continuous at $x = a$?
3. State the definition of derivative at a point. In other words, what is the defintion of the derivative of the function $f(x)$ at the point $x = a$?
4. State an alternative defintion for $f'(a)$.
5. State the definition of the derivative in general. That is, what is the defintion of the derivative $f'(x)$ of $f(x)$.
6. What is the relationship between differentiability and continuity? Does one guarantee the other? Are they equivalent? Give examples.
7. If $f'(a) = 0$, what can you say about $f(a)$? Be precise.
8. If $f'(a) > 0$, what can you say about f near a?
9. If $f'(a) < 0$, what can you say about f near a?
10. If $f''(a) > 0$, what can you say about f near a?
11. If $f''(a) < 0$, what can you say about f near a?
12. Carefully state the definitions of horizontal asymptote and vertical asymptotes. How do you find vertical asymptotes?
13. State the intermediate value theorem.
14. State the squeeze theorem.
15. State the product rule, the quotient rule, and the chain rule.
16. List all of the derivative formulas for the elementary functions. Memorize them.
17. State the extreme value theorem.
18. What is the closed interval method?
19. What is the first derivative test? the second derivative test?
20. Do the Concept Check on page 258.
21. Do the Concept Check (1–6 only) on page 336.

Other Topics

- parametric curves
- implicit differentiation
- logarithmic differentiation
- linear approximation
- related rates

Practice Problems

Find the following limits.

1. $\lim_{x\to -4} \dfrac{x^2+7x+12}{x+4} =$

2. $\lim_{x\to \infty} \ln x =$

3. $\lim_{x\to \infty} \tan^{-1} x = \lim_{x\to \infty} \arctan x =$

4. $\lim_{x\to 0^+} \ln x =$

5. $\lim_{x\to \infty} \dfrac{(2x+1)^3}{3x^3+1} =$

6. $\lim_{x\to -\infty} \dfrac{x+1}{\sqrt{x^2+1}} =$

7. $\lim_{x\to 2} |x-2| =$

8. $\lim_{x\to 2} \dfrac{|x-2|}{x-2} =$

9. $\lim_{h\to 0} \dfrac{e^h-1}{h} =$

10. $\lim_{h\to 0} \dfrac{(2+h)^{10}-1024}{h} =$

11. Compute $f'(x)$ if

(a) $f(x) = 7x^2 - \dfrac{1}{x^2}$

(b) $f(x) = 456$

(c) $f(x) = (x - e^x)\sqrt{x}$

(d) $f(x) = \dfrac{e^x\sqrt{x}}{x+1}$

(e) $f(x) = e^x + x^e$

12. Suppose that $f(x) = \dfrac{x^2+1}{x^2-1}$. Without using a calculator, complete the following:

(a) What is the domain of f?

(b) What are the roots of f?

(c) Compute all asymptotes of f?

(d) Find $f'(x)$ and $f''(x)$. This can be done with just the quotient rule.

(e) Carefully sketch a graph of f making sure that you include all important features of the graph.

13. Page 259 #1–#30

14. Page 259 #53

15. Page 337 #13

16. Page 338 #35

17. Consider the function $f(x) = e^{2x}$.

(a) Determine the linearization $L(x)$ of $f(x)$ at the point $(0, 1)$.

(b) Use your result in **(a)** to approximate $e^{0.2}$.

18. The equation $y^5 + xy^2 + x^3 = 4x + 3$ defines y implicitly as a function of x near the point $(2, 1)$.

(a) Determine the values of y' and y'' at this point.

(b) Use the linear (tangent line) approximation to estimate y when $x = 1.97$.

(c) Make a sketch showing how the curve relates to the tangent line near the point $(2, 1)$.