Math 111.01 Summer 2003
Assignment #5

This assignment is due at the beginning of class on **Friday, July 25, 2003**. You are encouraged to form study groups and collaborate with others on this assignment. However, the final work you submit must be your own. You must submit all problems that are marked with an asterix (*). YOUR ASSIGNMENT MUST BE STAPLED AND PROBLEM NUMBERS CLEARLY LABELLED. UNSTAPLED ASSIGNMENTS WILL NOT BE ACCEPTED!

1. Rework all the problems on Prelim #2, paying special attention to those that you got incorrect (even partially).

2. Practice problems.

- Section 4.5 #1, 3, 5, 9, 27, 41
- Section 4.6 #3, 7, 9, 15, 17, 27
- Section 4.8 # 1, 3, 9, 19, 21

3. * Problems to hand in.

- Section 4.5 #4, 10, 14, 34, 44, 54
- Section 4.6 #4, 8, 10, 12, 16, 32
- Section 4.8 #4, 6, 16, 20, 24

4. * On Assignment #3 you investigated $\lim_{x\to 0}(\sec x)^{1/x^2}$. Carefully show that the true value of this limit is

$$\lim_{x\to 0}(\sec x)^{1/x^2} = \sqrt{e}.$$

5. * Using L'Hôpital's Rule, find

$$\lim_{x\to a}\left(\frac{\sin x}{\sin a}\right)^{1/(x-a)}$$

where $\sin a \neq 0$.

6. * (Newton's Method)

(a) Show that Newton's method applied to the equation $x^3 + 3x - 2k = 0$ yields the iteration

$$x_{n+1} = \frac{2}{3}\frac{x_n^3 + k}{x_n^2 + 1}.$$

(b) Use this iteration to find the roots of $x^3 + 3x - 2 = 0$ accurate to five decimal places. (*Hint:* Start with $x_0 = 1$.)

(continued)

7. * Let $f(x) = x \ln x - 2$.

(a) Show that there exists a solution of $f(x) = 0$ between $x = 0$ and $x = e$. (*Hint:* Use a theorem from before Prelim 1!)

(b) Use Newton's method with starting value $x_0 = 2$ to find a solution to 6 decimal places.

(c) Show that the x_n in Newton's method are all bigger than the actual solution.