Math 111.01 Summer 2003
Assignment #1 Solutions

1. Practice problems.

Solutions may be found in the back of the text, or in the *Student Solutions Manual.*
Section 1.4 #2 Answer: (d) $[-2, 10] \times [-2, 6]$

2. Problems to hand in.

Section 1.1

#8. It is a function. Its domain is $[-3, 2]$ and its range is $-2 \cup (0, 3]$. The $\cup$ symbol means "union," which means that the range consists of -2 together with all values of y such that $0 < y \leq 3$.

#22.

$$f(2+h) = \frac{2+h}{2+h+1}$$

$$f(x+h) = \frac{x+h}{x+h+1}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h} = \frac{1}{(1+x)(1+h+x)}$$

#42.

$$f(x) = \begin{cases} 2, & \text{if } x \leq 0, \\ 2-2x, & \text{if } 0 < x \leq 1, \\ x-1, & \text{if } x > 1. \end{cases}$$

Section 1.2

#2. **a**. rational, algebraic

b. algebraic

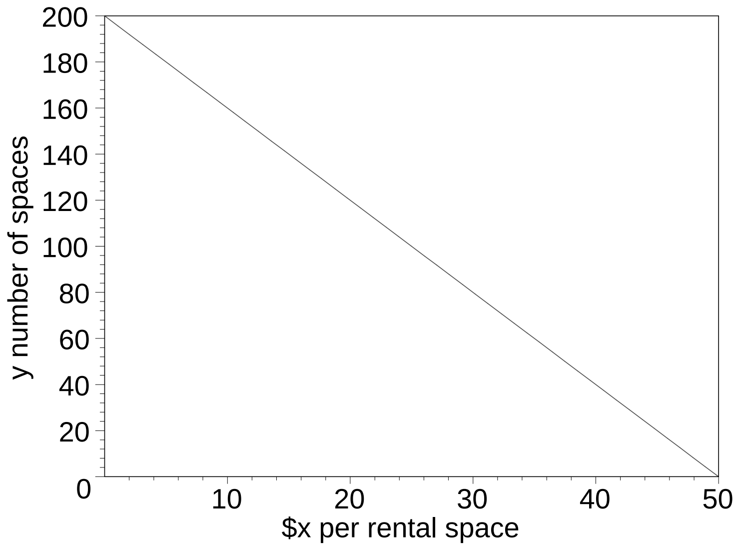
c. exponential

d. power, polynomial of degree 10, algebraic

e. polynomial of degree 6, algebraic

f. trigonometric

#6. **a**. $y = 200 - x$



b. The slope means that as the price per space *increases* by $1, the number of spaces the manager will be able to rent *decreases* by $4. The y-intercept of 200 is the number of spaces that would be occupied if they were rented for free. The x-intercept of $50 is the lowest price that will results in no spaces being rented (e.g., if the price was $60 per space, there would also be no spaces rented).

Section 1.3

#2. **a**. Stretch the graph vertically by a factor of 5.

b. Shift the graph to the right by 5.

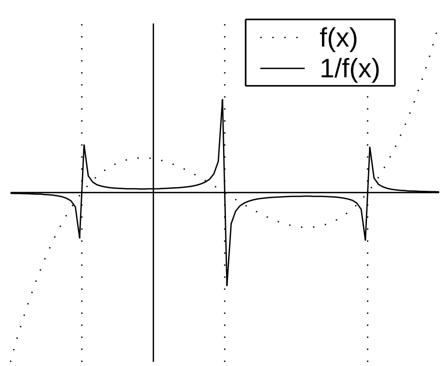
c. Reflect the graph over the x-axis.

d. Stretch the graph vertically by a factor of 5 and reflect it over the x-axis.

e. Compress the graph horizontally by a factor of 5.

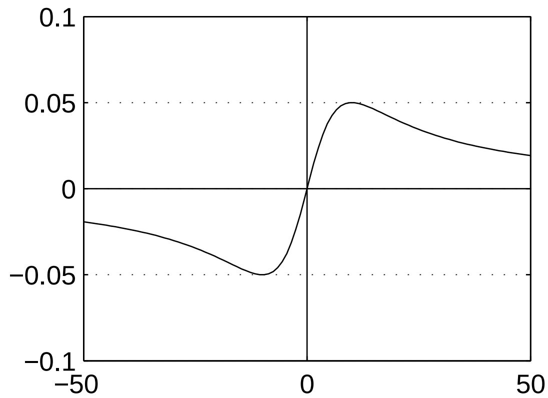
f. Stretch the graph vertically by a factor of 5 and move it down by 3.

#28. Where $f(x)$ has zeros, $1/f(x)$ will have vertical asymptotes. Where $f(x)$ is large, $1/f(x)$ will be small. $1/f(x)$ will be positive where $f(x)$ is positive and negative where $f(x)$ is negative. $1/f(x)$ will be 1 at $x = 0$ because $f(0) = 1$. So we have the picture:

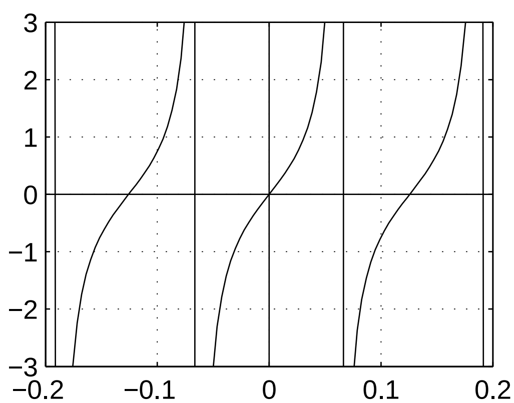


Section 1.4

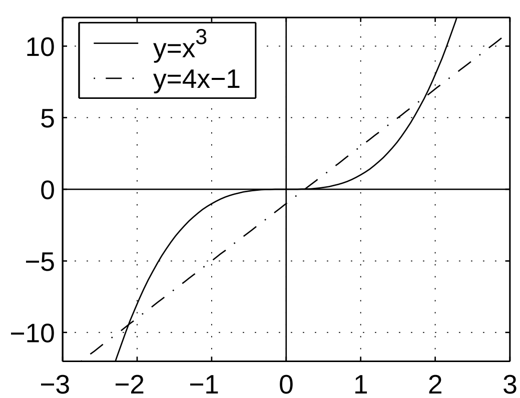
#8. $f(x) = \dfrac{x}{x^2 + 100}$



#12. $y = \tan(25x)$



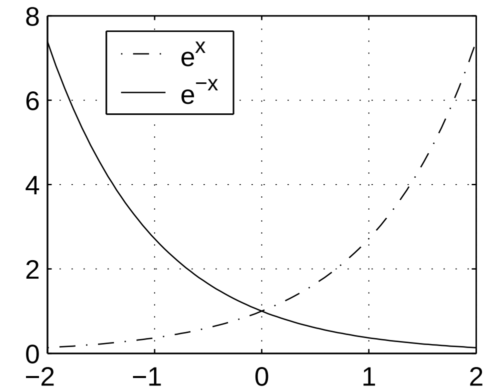
#18. $x^3 = 4x - 1$



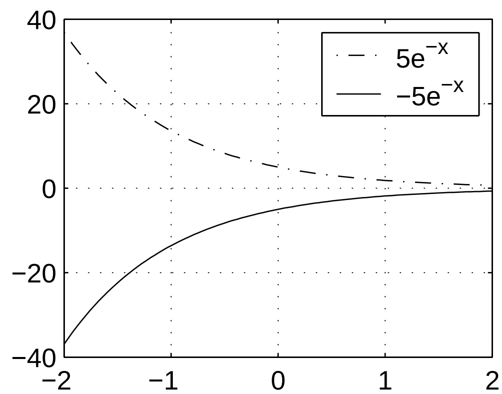
There are 3 intersection points of the two curves $y = x^3$ and $y = 4x - 1$ above, which are approximately -2.1, 0.2, and 1.8.

Section 1.5

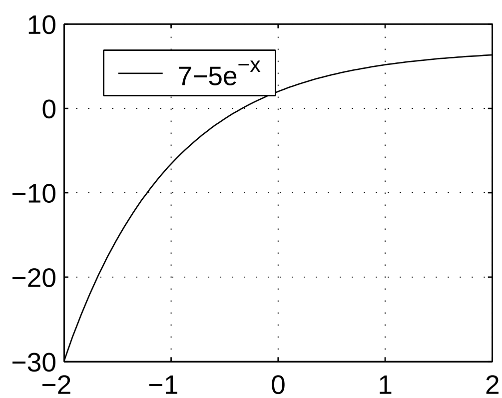
#12. First, simplify the equation to get $y = 7 - 5e^{-x}$. We begin by flipping e^x over the y-axis to get a graph of e^{-x}.



Then we stretch the graph vertically by a factor of 5, and then flip it over the x-axis to get a graph of $-5e^{-x}$.



Finally, we shift it up by 7 to obtain the graph of $y = 7 - 5e^{-x}$.



#18. You would prefer payment method II. Suppose the month is February (in order to pick the shortest month). Using method II your payment on the 28^{th} day would be $2^{28-1} = 2^{27} = 134217728$ cents or \$1,342,177.28 which is clearly better than method I.

Section 1.6

#10. The function $f(x) = 1+4x-x^2$ is not one-to-one because it is a parabola and $f(2+\sqrt{5}) = 0 = f(2-\sqrt{5})$ even though $2+\sqrt{5} \neq 2-\sqrt{5}$.

#22. We begin with

$$\begin{aligned} m &= \frac{m_0}{\sqrt{1-v^2/c^2}} \\ \sqrt{1-v^2/c^2} &= m_0/m \end{aligned}$$

Squaring both sides then gives

$$\begin{aligned} 1-v^2/c^2 &= m_0^2/m^2 \\ c^2-v^2 &= c^2m_0^2/m^2 \\ v^2 &= c^2-c^2m_0^2/m^2 \end{aligned}$$

Finally, we take the square root of both sides to give us

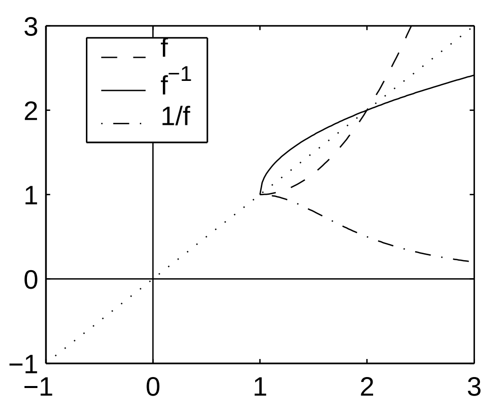
$$v = \sqrt{c^2-c^2m_0^2/m^2} = c\sqrt{1-m_0^2/m^2}.$$

Note that we only take the positive square root because the physical meaning of the equation is to give us the velocity of the particle as a function of its mass.

#28. Follow the series of manipulations:

$$\begin{aligned} y &= \frac{1+e^x}{1-e^x} \\ y(1-e^x) &= 1+e^x \\ y-ye^x &= 1+e^x \\ y-1 &= ye^x+e^x \\ y-1 &= (y+1)e^x \\ \frac{y-1}{y+1} &= e^x \\ x &= \ln\left(\frac{y-1}{y+1}\right) \Rightarrow y^{-1} = \ln\left(\frac{x-1}{x+1}\right) \end{aligned}$$

#32. To graph f^{-1}, we flip over the diagonal line $y = x$. To graph $1/f(x)$, we use the fact that as $f(x)$ gets large, $1/f(x)$ gets small.



Section 2.1

#2. **a.**

$$\text{slope} = \frac{2948 - 2530}{42 - 36} = \frac{418}{6} \approx 69.67$$

b.

$$\text{slope} = \frac{2948 - 2806}{42 - 40} = \frac{142}{2} = 71$$

c.

$$\text{slope} = \frac{2948 - 2661}{42 - 38} = \frac{287}{4} = 71.75$$

d.

$$\text{slope} = \frac{3080 - 2948}{44 - 42} = \frac{132}{2} = 66$$

The patient's heart rate is decreasing from 71 to 66 heart beats/minute after 42 minutes. After being stable for a while, the patient's heart rate is dropping.

#8. **a.** (i)

$$\text{average velocity} = \frac{s(5) - s(2)}{5 - 2} = \frac{178 - 32}{3} = \frac{146}{3} \approx 48.7 \text{ ft/s}$$

(ii)

$$\text{average velocity} = \frac{s(4) - s(2)}{4 - 2} = \frac{119 - 32}{2} = \frac{87}{2} = 43.5 \text{ ft/s}$$

(iii)

$$\text{average velocity} = \frac{s(3) - s(2)}{3 - 2} = \frac{70 - 32}{1} = 38 \text{ ft/s}$$

b. Using the points $(0.8, 0)$ and $(5, 118)$ from the approximate tangent line, the instantaneous velocity at $t = 2$ is about

$$\frac{118 - 0}{5 - 0.8} \approx 28 \text{ ft/s}.$$

3.

Let $f(t) = \log t$, $g(t) = \sqrt{t}$, and $h(t) = 1 - t$.

(a) $\mathscr{D}(\log t) = \{t > 0\} = (0, \infty)$, $\mathscr{R}(\log t) = \mathbb{R} = (-\infty, \infty)$

(b) $\mathscr{D}(\sqrt{t}) = \{t \geq 0\} = [0, \infty)$, $\mathscr{R}(\sqrt{t}) = \{t \geq 0\} = [0, \infty)$

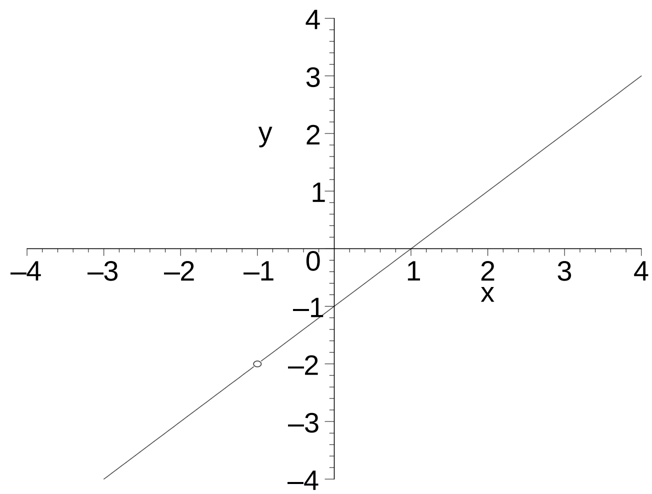
(c) $\mathscr{D}(1 - t) = \mathbb{R}$, $\mathscr{R}(1 - t) = \mathbb{R}$

(d) $\mathscr{D}(\log \sqrt{t}) = \mathscr{D}(\frac{1}{2} \log t) = \{t > 0\} = (0, \infty)$, $\mathscr{R}(\log \sqrt{t}) = \mathscr{R}(\frac{1}{2} \log t) = \mathbb{R}$

(e) $\mathscr{D}(\sqrt{1 - t}) = \{1 - t \geq 0\} = \{t \leq 1\} = (-\infty, 1]$, $\mathscr{R}(\sqrt{1 - t}) = \{t \geq 0\} = [0, \infty)$

(f) $\mathscr{D}(\log(\sqrt{1 - t})) = \mathscr{D}(\frac{1}{2} \log(1 - t)) = \{1 - t > 0\} = \{t < 1\} = (-\infty, 1)$, $\mathscr{R}(\log(\sqrt{1 - t})) = \mathbb{R}$

4. $f(x) = \frac{x^2-1}{x+1} = \frac{(x-1)(x+1)}{x+1}$. Thus, $\mathscr{D}(f) = \{x \neq -1\}$ and $\mathscr{R}(f) = \{f(x) \neq 2\}$.



The key observation here is that $f(x)$ is NOT defined at $x = -1$. Provided $x \neq -1$, we can divide the common factor, or "cancel out," to get

$$f(x) = \frac{(x-1)(x+1)}{x+1} = x - 1.$$

This means that the function $f(x)$ looks like the straight line $x - 1$, except that there is no defined value for $f(x)$ at $x = -1$.