Math 111.17 Fall 2002
Quiz #9 Solutions

1. By the definition of definite integral,

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\sqrt{1+\frac{k}{n}} \;=\; \int_1^2 \sqrt{x}\,dx \;=\; \frac{2}{3}\,x^{3/2}\Big|_1^2 \;=\; \frac{2}{3}(2^{3/2}-1).$$

2. Using the properties of definite integrals, we have

$$3\int_0^4 \cos^2 x\,dx + \int_0^2 3\sin^2 x\,dx - \int_2^4 3\cos^2 x\,dx = 3\int_0^4 \cos^2 x\,dx - \int_2^4 3\cos^2 x\,dx + \int_0^2 3\sin^2 x\,dx$$
$$= 3\int_0^2 \cos^2 x\,dx + 3\int_0^2 \sin^2 x\,dx = 3\int_0^2 \cos^2 x + \sin^2 x\,dx$$
$$= 3\int_0^2 1\,dx = 3\cdot(2-0) = 6$$

3. Since an antiderivative of a^x for $a>0$ is $\frac{1}{\ln a}a^x$, by the Evaluation Theorem,

$$\int_8^9 2^x\,dx \;=\; \frac{1}{\ln 2}\,2^x\Big|_8^9 \;=\; \frac{1}{\ln 2}(2^9-2^8) \;=\; \frac{2^8}{\ln 2}.$$

4. By the Fundamental Theorem of Calculus and the Chain Rule,

$$f'(x) = \frac{d}{dx}\int_{1.73\pi}^{\sqrt{x}} \frac{\cos t}{t}\,dt = \frac{\cos\sqrt{x}}{\sqrt{x}}\cdot\frac{d}{dx}\sqrt{x} = \frac{\cos\sqrt{x}}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}} = \frac{\cos\sqrt{x}}{2x}.$$

5. After splitting the numerator across the sum, recognizing the first integral, and using a simple substitution for the second ($u = 1+x^2$, $du = 2\,x\,dx$), we have

$$\int \frac{1+x}{1+x^2}\,dx = \int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx = \tan^{-1}x + \frac{1}{2}\ln(1+x^2) + C.$$