Math 111.17 Fall 2002
October 9, 2002

Exercise: Suppose $xy + \sin(x + y) = 3$. Compute y' and y''.

Solution: We do this implicitly:

$$\begin{aligned} \frac{d}{dx}(xy) + \frac{d}{dx}(\sin(x+y)) &= \frac{d}{dx}(3) \\ y\frac{dx}{dx} + x\frac{dy}{dx} + \cos(x+y)\cdot\frac{d}{dx}(x+y) &= 0 \\ y + x\frac{dy}{dx} + \cos(x+y)\cdot(1+\frac{dy}{dx}) &= 0 \end{aligned}$$

We now gather terms with $\frac{dy}{dx}$, and factor.

$$\begin{aligned} x\frac{dy}{dx} + \cos(x+y)\frac{dy}{dx} &= -y - \cos(x+y) \\ \frac{dy}{dx} &= \frac{-y-\cos(x+y)}{x+\cos(x+y)} \end{aligned}$$

Thus,

$$\boxed{y' = \frac{-y-\cos(x+y)}{x+\cos(x+y)}.}$$

In order to compute y'', we use the quotient rule. Therefore,

$$\begin{aligned} y'' &= \frac{\frac{d}{dx}(-y-\cos(x+y))(x+\cos(x+y)) - \frac{d}{dx}(x+\cos(x+y))(-y-\cos(x+y))}{(x+\cos(x+y))^2} \\ &= \frac{(-y' + \sin(x+y)\cdot(1+y'))(x+\cos(x+y)) - (1-\sin(x+y)\cdot(1+y'))(-y-\cos(x+y))}{(x+\cos(x+y))^2} \end{aligned}$$

Since we know y' we can substitute that into the above.

[Please, please do not simplify.]