Math 111.17 Fall 2002
November 6, 2002

1. You want to make a rectangular box with square base and open top that will have a volume of 270 cubic inches. The material for the bottom of the box costs 6 cents per square inch and the material for the sides costs 3 cents per square inch. Find the dimensions of the box that will minimize your total cost.

2. Evilcorp, the company you work for, wants to distribute millions of copies of a page of their advertising propaganda. The page is to contain 24 square inches of printed material, with margins of 1.5 inches at the top and bottom of the page, and 1 inch at the sides. Since paper is expensive, Evilcorp wants to minimize the amount of paper needed to print such a page. Find the dimensions (length and width) of the page that will minimize the total area.

3. For her birthday, your mother wants a painted jewelry box with a square base and a volume of 96 cubic inches. If the top of the box costs 6 cents per square inch to paint, and painting the sides and base of the box costs 3 cents per square inch, what dimensions of the box will minimize the total cost of painting?

4. Your rectangular box project (from problem 1) was so successful that you want to make a series of boxes with the same construction, but with varying volumes. Determine the dimensions that minimize the total cost in terms of V, the volume of the box in cubic inches.

5. Evilcorp tells you that the margin sizes they told you before are wrong (see problem 2), but that they haven't figured out the new ones yet. Find the overall dimensions of the page that minimize the total area of paper, in terms of T and S, the top/bottom and left/right margin lengths.

6. Waste Management, Inc. makes small, heavy-duty cans (which are cylindrical with closed top and bottom, of course) out of two kinds of material. The material used to make the top and bottom of the can costs 6 cents per square inch, and the material used to make the curved surface of the can costs 3 cents per square inch. The company is willing to pay \$1.44 for the material used in each can, but wants the can to hold as much waste as possible. If r is the radius and h is the height of the can, find the value of r that will maximize the amount of waste the can will be able to hold.

7. A rectangular box with square base and open top is constructed from two types of material. The material used to make the bottom of the box costs 10 cents per square inch, and the material used to make the rest of the box costs 6 cents per square inch. The total cost of the box is \$3.00. If s is one side of the base of the box and h is the height, find the value of s that maximizes the volume V.

8. A wire 6 meters long is cut into twelve pieces, eight of one length and four of another. These pieces are welded together at right angles to form the frame of a box with a square base.

(a) Where should the wire be cut to maximize the volume of the box?

(b) Where should the cuts be made to maximize the total surface area of the box?

9. Being short on cash but big on land, you divide up the land behind your house into 200 plots and rent the space to local gardeners. When you charge \$250 per season (per plot), all of your plots get rented. For every \$10 increase in rent, 5 of your plots go unrented (gardeners don't have that much money either). What rent should you charge to obtain the most amount of money in a season?

10. Waste Management, Inc. (see problem 6) makes another kind of can, one with a closed bottom but an open top. The material used to make the bottom of the can costs 5 cents per square inch, and the material used to make the curved surface of the can costs 3 cents per square inch. If the total cost of the can is to be no more than \$7.35, what dimensions will give the can the greatest volume?

11. The Transcam company has just landed a big contract for steel wool. Because steel wool is relatively light, the shipping department wants to pack as much as possible into each box sent out to cut down on handling charges. The trucking company allows boxes to have a combined length and girth (the distance measured around the box perpendicular to its length) of no more than 108 inches. If the shipping department uses boxes with square ends, what box dimensions will maximize the volume the box can hold?

12. Suppose that Transcam (see problem 11) decides to ship its steel wool in cylindrical containers instead of boxes.

(a) What height and radius will maximize the volume of the cylindrical containers?

(b) Which is the better choice for Transcam, the box or the cylindrical container?

Do each of the following problems algebraically first, then use your calculator (when possible) to check your answers. Use the first or second derivative test to determine whether your critical points are local maxima or minima (or neither), and be sure to test the "endpoints" of the domain.

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13. A rectangular plot of farmland is to be bounded on one side by a river and on the other three sides by a single-strand electric fence.

(a) With 900 feet of wire at your disposal, what is the largest area you can enclose?

(b) If you have L feet of wire at your disposal, what is the largest area you can enclose?

14. Suppose you are designing an oil can shaped like a right circular cylinder, and you need the can to have a volume of 100 cm^3.

(a) What dimensions will use the least material?

(b) If the can has to have a volume of V cm^3, what dimensions will use the least material?

15. An 8-inch wire is to be cut into two pieces. One of the pieces will be formed into a circle, and the other will be formed into a square.

(a) How should the wire be cut in order to minimize the sum of the enclosed areas?

(b) If the wire is l inches long, what is this minimum area that can be enclosed as above?

16. A fabric manufacturer must supply a customer with 12,000 yards of cloth during the coming year. The fabric will be produced in batches throughout the year, and stored in the manufacturer's warehouse. The manager wants to produce the same number of yards at each production run. Suppose that:

- The set-up cost per batch is \$300.
- The storage cost per year is \$0.80 times the average number of yards in stock.
- The production cost is \$3.00 per yard.

Also suppose that the average number of yards in stock is about half the number of yards produced in a batch. (Why does this make sense?)

(a) How many yards of fabric should be made during each production run to minimize the total cost of producing 12,000 yards of fabric? Make sure that you take into account the fact that the cloth will be produced in batches when you write the total cost function.

(b) Explain in terms of the problem why it is or is not optimal for the fabric manufacturer to produce all of the fabric at once.

17. A farmer owns 0.25 square miles of land along a straight river, and wishes to divide it into eight rectangular plots by putting up one fence parallel to the river and nine fences perpendicular to it.

(a) What is the least amount of fence the farmer can use to enclose these plots?

(b) What is the least amount of area the farmer can enclose as above?