Math 111 Fall 2002
Linear Approximation Practice Problems

Since there were no problems on linear approximation on the second practice prelim, we are including some separately.

1. Consider the function $f(x) = e^{2x}$.

(a) Determine the linearization $L(x)$ of $f(x)$ at the point $(0, 1)$.

(b) Use your result in **(a)** to approximate $e^{0.2}$.

(c) For which values of x is $L(x)$ accurate to within 0.1 of $f(x)$?

2. The equation $y^5 + xy^2 + x^3 = 4x + 3$ defines y implicitly as a function of x near the point $(2, 1)$.

(a) Determine the values of y' and y'' at this point.

(b) Use the linear (tangent line) approximation to estimate y when $x = 1.97$.

(c) Make a sketch showing how the curve relates to the tangent line near the point $(2, 1)$.

3. One use of a linear approximation is to prove the chain rule. Recall that the definition of the derivative of the function f is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Use a linear approximation to prove the chain rule. That is, prove that

$$[f(g(x))]' = f'(g(x)) \cdot g'(x).$$

Hint: Differentiate $f(g(x))$ using $g(x+h) \approx g(x) + hg'(x)$ and $f(z+k) \approx f(z) + kf'(z)$.