Math 105 Prelim #3 – November 20, 2003

This exam has 7 problems and 7 numbered pages.

You have 90 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

Calculators are permitted, but no other aids are allowed.

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name: ______________________________

Instructor: ______________________________

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TOTAL: __________

Bayes' Theorem (as stated on page 366 of the textbook)

$$P(F_i|E) = \frac{P(F_i)P(E|F_i)}{P(F_1)P(E|F_1) + P(F_2)P(E|F_2) + \cdots + P(F_n)P(E|F_n)}$$

Standard Deviation (as stated on page 455 of the textbook)
The standard deviation of the n numbers $x_1, x_2, x_3, \ldots, x_n$ with mean $\overline{x}$ is

$$s = \sqrt{\frac{\sum x_i^2 - n(\overline{x})^2}{n-1}}.$$

Mean and Standard Deviation for Binomial Distribution(as stated on page 477 of the textbook)
For the binomial distribution, the mean and standard deviation are given by

$$\mu = np \qquad \sigma = \sqrt{np(1-p)}.$$

The final examination will be held on **Thursday, December 11, 2003** at **9:00 a.m.** in **Warren Hall (WN) B45**.

Please sign below acknowledging that you have read all of the prelim instructions and the final examination information.

(signed) ____________________________________

1. (*20 points*) A geneticist counted the number of bristles on a certain region of the abdomen of the fruitfly *Cortlandophilia melanogaster*. The results for 67 individual fruitflies are shown in the table below.

# of bristles	33	34	35	36	37	38	39
# of flies	2	2	9	11	12	18	13

(a) Determine the mode number of bristles on *Cortlandophilia melanogaster* based on this data.

(b) Compute the median number of bristles on *Cortlandophilia melanogaster* based on this data.

(c) Compute the mean number of bristles on *Cortlandophilia melanogaster* based on this data.

(d) Compute the standard deviation of the number of bristles on *Cortlandophilia melanogaster* based on this data.

2. (*15 points*) A Cortland County farm store sells bags of grapes. Assume that the weight of grapes in each bag is normally distributed with a mean of 16 ounces and a standard deviation of 0.5 ounces.

(a) If you randomly buy one of these bags of grapes, what is the probability that you will get a bag that weighs more than 17 ounces?

(b) If you randomly buy three of these bags of grapes, what is the probability that at least one of them will weigh more than 17 ounces?

(c) Find the weight in ounces so that 90% of this Cortland County farm store's bags of grapes are heavier than this weight?

3. (*18 points*) The Cortland Post Office reports that the probability that first class mail sent on Monday will be delivered on Tuesday is 0.90. In an experiment, you post one letter to each of 10 friends around the country on Monday.

(a) What is the probability that exactly 9 of your friends receive your letter on Tuesday?

(b) How many of your friends do you expect to receive your letter on Tuesday?

(c) What is the probability that less than 8 of your friends receive your letter on Tuesday? *(In other words, what is the probability that at most 7 of your friends receive your letter on Tuesday?)*

4. (*12 points*) For each of the following pairs, determine whether $p < q$ or $p > q$ or $p = q$.

(i) A standard six-sided die is rolled six times: $p = P(3 \text{ ones})$ and $q = P(4 \text{ twos})$.

(ii) Five cards are drawn at random from a standard deck of 52 cards with replacement: $p = P(\text{more than 3 face cards})$ and $q = P(\text{less than 2 red cards})$. *(Note that face cards are Jack, Queen, or King, and the red cards are hearts and diamonds.)*

(iii) A fair coin is flipped ten times: $p = P(n \text{ heads})$ and $q = P(10 - n \text{ heads})$ where $0 \leq n \leq 10$.

5. (*15 points*) The Cortland County Lottery is played as follows. You select a four digit number by choosing four **different** numbers from $\{0, 1, 2, \ldots, 9\}$. *(For example, 0923 is allowed, but 1282 is not allowed.)*

The Cortland Lottery Corporation selects the winning four digit number by successively drawing without replacement from an urn containing ten balls labelled $0, 1, 2, \ldots, 9$, repectively.

If your four digit number matches the Lottery Corporation's winning number *in order*, then you win a prize of \$500. *(For example, 0923 and 9032 do not match.)*

(a) What is the probability of winning a prize of \$500?

(b) If each lottery ticket costs \$2, what is your expected profit when playing this lottery?

A game is called **fair** if your expected profit is exactly \$0.

(c) Is this lottery a fair game?

(d) If the ticket price were to remain at \$2 but the prize amount could be changed, what prize amount would result in this being a fair game?

6. (*10 points*) 1% of the Cortland County population has a mutation in their DNA that gives them a 10% chance of gaining superpowers. The rest of the Cortland County population has only a 4% chance of gaining superpowers. If, in one group of seven identical Cortland County septuplets, 4 gain superpowers, what is the probability that they have the mutated DNA? *(Note that we are assuming that all seven septuplets have identical DNA.)*

7. (*10 points*) You have a fair six-sided die which has three sides painted yellow, two sides painted red, and one side painted green. Suppose that you roll this die 98 times. Let Y be a random variable representing the total number of upturned red faces observed on these 98 rolls. Note that the possible values of Y are $0, 1, 2, \ldots, 97, 98$.

(a) What is $E(Y)$, the expected value of Y?

(b) Using the normal approximation to the binomial, what is $P(30 \leq Y \leq 40)$, the probability that Y is between 30 and 40 (inclusive)?