Math 105 Prelim #2 – Solutions

1. By definition,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

provided that $P(B) > 0$ (which it is, in this case). Since $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ we find that

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.38 + 0.42 - 0.59 = 0.21.$$

Thus,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.21}{0.42} = \boxed{\frac{1}{2}.}$$

2. The order of the couples does not matter, only that the boy-girl couples be different. There are 12 boys that the first girl could pick, leaving 11 boys that the second girl could pick, etc. Thus there are

$$\boxed{12 \times 11 \times \cdots \times 2 \times 1 = 12!}$$

ways they can pair off. (Alternatively, the boys could pick the girls, but that would result in the same boy-girl couples as above.)

3.

(a) In order to pick the second ball from the blue box, a blue ball must have been drawn on the first ball picked from the grey box. Let B be the event that blue is drawn from the grey box, and let S be the sample space consisting of all balls in the grey box. Then

$$P(B) = \frac{n(B)}{n(S)} = \boxed{\frac{2}{3}.}$$

(b) Since the first ball is blue, we must pick a ball from the blue box. Let N (for noir) be the event that black is drawn from the blue box, and let T be the sample space consisting of all balls in the blue box. Then

$$P(N|B) = \frac{n(N)}{n(T)} = \boxed{\frac{4}{5}.}$$

(c) Let R be the event that red is drawn from the grey box, so that (as in **(a)**) $P(R) = 1/3$ and that (similar to **(b)**) $P(N|R) = 1/5$. Thus,

$$P(N) = P(N|B)P(B) + P(N|R)P(R) = \frac{4}{5} \cdot \frac{2}{3} + \frac{1}{5} \cdot \frac{1}{3} = \boxed{\frac{9}{15}}.$$

(d) By the definition of conditional probability (or by Bayes' Theorem),

$$P(B|N) = \frac{P(N|B)P(B)}{P(N)} = \frac{4/5 \times 2/3}{9/15} = \boxed{\frac{8}{9}}.$$

4.

(a) $\boxed{\textbf{II}}$ If $P(E \cup F) = P(E) + P(F)$, then $P(E \cap F) = 0$. But we are told that $P(E) > 0$ and $P(F) > 0$ so that $P(E)P(F) > 0$. Thus, E and F cannot be independent.

(b) $\boxed{\textbf{III}}$ The definition of conditional probability says that if $P(F|E) = P(F)$, then E and F are independent. However, it does not say anything about $P(F|E) = P(E)$. They could still be independent, or they could not be independent.

(c) $\boxed{\textbf{I}}$ This is the definition of conditional probability. If $P(F|E) = P(F)$, then E and F are independent.

(d) $\boxed{\textbf{I}}$ Since $P(E)P(F) = 0.2 \times 0.5 = 0.1 = P(E \cap F)$, E and F are independent.

(e) $\boxed{\textbf{II}}$ Since $P(E)P(F) = 0.3 \times 0.2 = 0.6 \neq 0.1 = P(E \cap F)$, E and F are not independent.

5. Let A, B, C be the events that the manufacturer buys the items from supplier A, B, C, respectively, and let D be the event that an item is defective. We want to find $P(A|D)$. By Bayes' Theorem,

$$\begin{aligned} P(A|D) = \frac{P(D|A)P(A)}{P(D)} &= \frac{P(D|A)P(A)}{P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C)} \\ &= \frac{0.05 \times 0.20}{0.05 \times 0.20 + 0.03 \times 0.50 + 0.07 \times 0.30} \\ &= \frac{0.010}{0.010 + 0.015 + 0.021} \end{aligned}$$

That is,

$$P(A|D) = \frac{0.010}{0.046} = \boxed{\frac{5}{23}}.$$

6.

(a) Since the events F_1, F_2, and F_3 are equally likely, we have $P(F_1) = P(F_2) = P(F_3)$. Since F_1, F_2, F_3 partition S, we have $P(F_1) + P(F_2) + P(F_3) = 1$ so that $3P(F_3) = 1$ or

$$P(F_3) = \boxed{\frac{1}{3}.}$$

(b) Since B is independent of F_3, we have BY DEFINITION OF INDEPENDENCE that

$$P(F_3|B) = P(F_3) = \boxed{\frac{1}{3}.}$$

Note that it is technically incorrect to write "since B is independent of F_3 we have $P(F_3 \cap B) = P(F_3)P(B)$ so that

$$P(F_3|B) = \frac{P(F_3 \cap B)}{P(B)} = \frac{P(F_3)P(B)}{P(B)} = P(F_3) = \frac{1}{3}\text{''}$$

because this requires that $P(B) \neq 0$. Otherwise, you would have divided by 0 in the first step. No such assumption was made in the problem.

(c) Since $P(F_3 \cap C) = P(C \cap F_3)$ we have that $P(F_3|C)P(C) = P(C|F_3)P(F_3)$ by the definition of conditional probability. *(Note that this was discussed in the solution to practice problem #0.)* Since $P(C') = 1/3$ we have that $P(C) = 1 - P(C') = 1 - 1/3 = 2/3$. Thus,

$$P(F_3|C) = \frac{P(C|F_3)P(F_3)}{P(C)} = \frac{1/4 \times 1/3}{2/3} = \boxed{\frac{1}{8}.}$$

7.

(a) There are 35 Kit-Kats of which she chooses 3, and there are 20 Almond Joys of which she chooses 1, and there are 22 Snickers of which she chooses 1. By the multiplication principle, this can be done in

$$\binom{35}{3} \times \binom{20}{1} \times \binom{22}{1}$$

ways. Since there are $\binom{77}{5}$ total ways she can grab 5 candy bars, the probability that she took 3 Kit-Kats, 1 Almond Joy, and 1 Snickers is

$$\boxed{\frac{\binom{35}{3} \times \binom{20}{1} \times \binom{22}{1}}{\binom{77}{5}}}.$$

(b) There are $22 + 20 = 42$ candy bars with nuts. Thus, she can choose 5 bars with nuts in $\binom{42}{5}$ ways so that the probability that she took 5 bars with nuts is

$$\boxed{\frac{\binom{42}{5}}{\binom{77}{5}}}.$$

(c) In order to choose 5 of the same kind of candy bars she could have chosen either 5 Kits-Kats, which could be done in $\binom{35}{5}$ ways, or she could have chosen 5 Almond Joys, which could have been done in $\binom{20}{5}$ ways, or she could have chosen 5 Snickers, which could have been done in $\binom{22}{5}$ ways. Thus, the probability that she took 5 of the same kind of candy bar is

$$\boxed{\frac{\binom{35}{5} + \binom{20}{5} + \binom{22}{5}}{\binom{77}{5}}}.$$

8.

(a) There are 9 ways to go from Ithaca to Elmira. For each of those ways, there are 4 ways to get from Elmira to Binghamton. And for each of those ways, there are 4 ways to get from Binghamton to Elmira. By the multiplication principle, there are

$$\boxed{9 \times 4 \times 4 = 144}$$

different travel routes.

(b) There are 9 ways to go from Ithaca to Elmira. For each of those ways, there are 4 ways to get from Elmira to Binghamton. And for each of those ways, there are only 3 ways to get from Binghamton to Elmira since you cannot travel the same route back. By the multiplication principle, there are

$$\boxed{9 \times 4 \times 3 = 108}$$

different travel routes if you do not want to travel the same route twice.

9.

(a) There are 6! ways to line up the six letters 'b', 'a', 'n', 'a', 'n', 'a'. However, there are 3! repetitions due to 'a', and there are 2! repetitions due to 'n'. Thus, there are

$$\boxed{\frac{6!}{3!2!} = 60}$$

different words that can be written.

(b) The only way to solve this problem is to write out the possibilities. Note that there are three 'a's and two 'n's and one 'b'.

Thus, if we want to use three 'a's there is one possibility, namely 'aaa'.

If we want to use two 'a's we can combine them with one 'n' in $3!/2! = 3$ ways, namely 'naa', 'ana', 'aan'.

If we want to use two 'a's we can also combine them with one 'b' in $3!/2! = 3$ ways, namely 'baa', 'aba', 'aab'.

If we want to use two 'n's we can combine them with one 'a' in $3!/2! = 3$ ways, namely 'ann', 'nan', 'nna'.

If we want to use two 'n's we can also combine them with one 'b' in $3!/2! = 3$ ways, namely 'bnn', 'nbn', 'nnb'.

Finally, if we want to use the lone 'b', then we must combine it with one 'a' and one 'n' (since all other possibilities have been accounted for). This can be done in $3! = 6$ ways, namely 'ban', 'bna', 'abn', 'anb', 'nba', 'nab'.

Thus, there are $\boxed{19}$ possible three letter words.