Math 105 Prelim #2 – October 30, 2003

This exam has 9 problems and 8 numbered pages.

You have 90 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

Calculators are permitted, but no other aids are allowed.

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name: ______________________________

Instructor: ______________________________

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TOTAL: ____________

Bayes' Theorem (as stated on page 366 of the textbook)

$$P(F_i|E) = \frac{P(F_i)P(E|F_i)}{P(F_1)P(E|F_1) + P(F_2)P(E|F_2) + \cdots + P(F_n)P(E|F_n)}$$

The final examination will be held on **Thursday, December 11, 2003** at **9:00 a.m.** in **Warren Hall (WN) B45**.

Please sign below acknowledging that you have read all of the prelim instructions and the final examination information.

(signed) ___________________________________

1. (*5 points*) Suppose that A and B are events with $P(A) = 0.38$, $P(B) = 0.42$, and $P(A \cup B) = 0.59$. Compute $P(A|B)$.

2. (*5 points*) There are 12 girls and 12 boys at the Ithaca Junior High School dance. How many ways can they pair off into 12 boy-girl partners?

3. (*20 points*) There are three boxes: one grey box, one blue box, and one red box. The grey box contains 2 blue balls and 1 red ball. The blue box contains 4 black balls and 1 white ball. The red box contains 1 black ball and 4 white balls. We pick a first ball from the grey box. If that first ball is blue, then we pick a second ball from the blue box. If that first ball is red, then we pick a second ball from the red box.

(a) What is the probability that the second ball is picked from the blue box?

(b) Given that the first ball is blue, what is the probability that the second ball is black?

(c) What is the probability that the second ball is black?

(d) Given that the second ball is black, what is the probability that it comes from the blue box (i.e., the first ball was blue)?

4. (*10 points*) In each of the following examples, determine if

(I) E and F are independent events, or

(II) E and F are not independent events, or

(III) it is unknown whether E and F are independent events.

(a) $P(E) > 0$, $P(F) > 0$, and $P(E \cup F) = P(E) + P(F)$.

(b) $P(E|F) = P(F)$.

(c) $P(F|E) = P(F)$.

(d) $P(E) = 0.2$, $P(F) = 0.5$, and $P(E \cap F) = 0.1$.

(e) $P(E) = 0.3$, $P(F) = 0.2$, and $P(E \cap F) = 0.1$.

5. (*10 points*) A manufacturer buys items from three suppliers: A, B, C. Supplier A provides 20% of the items, supplier B provides 50% of the items, and supplier C provides the remainder of the items. The probability that an item is defective is 0.05 if it comes from supplier A, 0.03 if it comes from supplier B, and 0.07 if it comes from supplier C. Suppose that the manufacturer receives a defective item. What is the probability that this defective item comes from supplier A?

6. (*15 points*) Suppose a sample space S is partitioned into three subsets F_1, F_2, F_3, each occurring with equal probability. That is, $F_1 \cup F_2 \cup F_3 = S$ with $F_1 \cap F_2 = \emptyset$, $F_1 \cap F_3 = \emptyset$, $F_2 \cap F_3 = \emptyset$, and $F_1 \cap F_2 \cap F_3 = \emptyset$.

(a) What is $P(F_3)$?

(b) If B is an event which is independent of F_1, F_2, and F_3, what is $P(F_3|B)$?

(c) If C is an event and $P(C|F_3) = \frac{1}{4}$, what is $P(F_3|C)$?

7. (*15 points*) You are handing out Hallowe'en candy to trick-or-treaters; you have a giant bowl filled with 22 Snickers, 35 Kit Kats, and 20 Almond Joy bars. A particularly hungry fairy princess grabs five candy bars at random from your bowl. What is the probability that she took:

(a) 3 Kit-Kats, 1 Almond Joy, and 1 Snickers?

(b) only candy with nuts? (Snickers and Almond Joy have nuts, Kit Kat does not.)

(c) five of the same kind of candy bars?

8. (*10 points*) You and your friends are planning a Thanksgiving break road trip, and want to travel from Ithaca to Elmira, then from Elmira to Binghamton, and finally from Binghamton back to Elmira. Suppose there are nine routes between Elmira and Ithaca, and four routes between Elmira and Binghamton.

(a) How many different travel routes are there?

(b) How many different travel routes are there if you do not want to go on the same road twice?

9. (*10 points*)

(a) How many different words can be written using all six letters of “banana”? (Assume that a word is a combination of letters, whether or not that word is in a dictionary.)

(b) How many 3 letter words can be written using the six letters of “banana”? (Again assume that a word is a combination of letters, whether or not that word is in a dictionary.)