Math 105 Prelim #1 – Solutions

1.

(a) $\boxed{\textbf{ii.}}$ m is negative, but we do not know the exact value of m

(b) $\boxed{\textbf{iii.}}$ most of the data points are close to the least squares line

2.

(a) Note that the independent variable x is the number of cases of EZ-CHEEZ produced, and the dependant variable $C = C(x)$ is the cost.

Since the cost function is assumed linear, and passes through the points $(10, 500)$ and $(20, 650)$, we find the slope to be

$$m = \frac{650 - 500}{20 - 10} = 15.$$

Using the point-slope form of a line, and (arbitrarily) selecting $(10, 500)$, we have $C(x) - 500 = 15(x - 10)$, so that

$$\boxed{C(x) = 15x + 350.}$$

Of course, we could have used the point $(20, 650)$ to obtain the same answer.

(b) Note that in this part, the independent variable x is the number of cases of EZ-CHEEZ produced, and the dependant variable $R = R(x)$ is the revenue.

Clearly,

$$\boxed{R(x) = 20x}$$

since each case sold earns us \$20.

(c) The break-even point occurs when the cost function and the revenue function intersect. This happens when $C(x) = R(x)$. Therfore, $15x + 350 = 20x$ or $5x = 350$ or $x = 70$, so that the break-even point is when

$$\boxed{\text{7 cases are sold.}}$$

3. We rewrite the system of equations as

$$\begin{aligned} -2x + 2y + 4z &= 0 \\ 4y + 6z &= 4 \\ 4x - z &= 5 \end{aligned}$$

so that we can form the augmented matrix

$$\left[\begin{array}{ccc|c} -2 & 2 & 4 & 0 \\ 0 & 4 & 6 & 4 \\ 4 & 0 & -1 & 5 \end{array}\right].$$

$2R_1 + R_3 \mapsto R_3$

$$\left[\begin{array}{ccc|c} -2 & 2 & 4 & 0 \\ 0 & 4 & 6 & 4 \\ 0 & 4 & 7 & 5 \end{array}\right]$$

$-R_1/2 \mapsto R_1$ and $-R_2 + R_3 \mapsto R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 0 & 4 & 6 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$-6R_3 + R_2 \mapsto R_2$ and $2R_3 + R_1 \mapsto R_1$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 0 & 4 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$4R_1 + R_2 \mapsto R_1$

$$\left[\begin{array}{ccc|c} 4 & 0 & 0 & 6 \\ 0 & 4 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Thus $4x = 6$, $4y = -2$, and $z = 1$ so that

$$\boxed{(x, y, z) = \left(\frac{3}{2}, -\frac{1}{2}, 1\right).}$$

4.

We attempt to row reduce the augmented matrix $[A|I]$ to $[I|A^{-1}]$. Thus,

$$[A|I] = \left[\begin{array}{cc|cc} 2 & -2 & 1 & 0 \\ 3 & -6 & 0 & 1 \end{array}\right].$$

Now, $-3R_1 + 2R_2 \mapsto R_2$ gives

$$\left[\begin{array}{cc|cc} 2 & -2 & 1 & 0 \\ 0 & -6 & -3 & 2 \end{array}\right].$$

Next, $R_2 - 3R_1 \mapsto R_1$ gives

$$\left[\begin{array}{cc|cc} -6 & 0 & -6 & 2 \\ 0 & -6 & -3 & 2 \end{array}\right].$$

Finally, $-R_2/6 \mapsto R_2$ and $-R_1/6 \mapsto R_1$ gives

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & -1/3 \\ 0 & 1 & 1/2 & -1/3 \end{array}\right].$$

Thus,

$$\boxed{A^{-1} = \begin{bmatrix} 1 & -1/3 \\ 1/2 & -1/3 \end{bmatrix}.}$$

5.

(a) The matrix product $\boxed{BC \text{ does not exist.}}$ Since B is 2×2 and C is 3×2, they **cannot** be multiplied. Only when the number of columns of the first matrix equals the number of rows of the second matrix can they be multiplied.

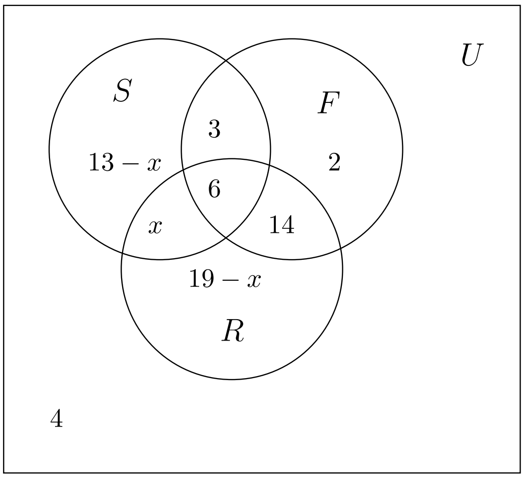
(b) $$CB = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1+0 & 2+0 \\ 1-3 & 2-4 \\ 2+3 & 4+4 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 2 \\ -2 & -2 \\ 5 & 8 \end{bmatrix}}$$

6.

The three parts of the problem are easily solved once we are able to fill in all 8 regions of the appropriate Venn diagram.

Let U be the universal set of all 50 cacti in the experiment, let S be the set of cacti that were spiny, let F be the set of cacti that had flowers, and let R be the set of cacti that produced sweet fruit.

We let x be the number of cacti that were both spiny and produced sweet fruit, but did not have flowers, as is shown in the diagram. We can then fill in the missing regions in terms of x using the given information.



Since 4 cacti had none of the traits, we see that $13 - x + 3 + 2 + x + 6 + 14 + 19 - x = 46$ so that $57 - x = 46$ or

$$x = 11.$$

(a) $\boxed{17}$ cacti were spiny and produced sweet fruit.

(b) $\boxed{2}$ cacti were spiny and had neither flowers nor produced sweet fruit.

(c) $\boxed{14}$ cacti were not spiny but had flowers and produced sweet fruit.

Note that with a change in wording, this is problem #41 on page 323.

7.

(a) If we write H and T for the two posssible outcomes on the flip of the coin, and 1, 2, 3, 4, 5, 6 for the 6 possible outcomes on the roll of the die, then

$$\boxed{S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}.}$$

Note that each outcome in S is equally likely.

(b) Let B be the event that you flip a head and roll a 6. Then $B = \{H6\}$, so

$$P(B) = \frac{n(B)}{n(S)} = \boxed{\frac{1}{12}.}$$

(c) Let C be the event that you flip a tail and roll an even number. Then $C = \{T2, T4, T6\}$, so

$$P(C) = \frac{n(C)}{n(S)} = \frac{3}{12} = \boxed{\frac{1}{4}.}$$

(d) Let D be the event that you either flip a tail or roll an even number. Then $D = \{T1, T2, T3, T4, T5, T6, H2, H4, H6\}$, so

$$P(D) = \frac{n(D)}{n(S)} = \frac{9}{12} = \boxed{\frac{3}{4}}.$$

(e) Let E be the event that you do not roll a 5. Then $E = \{H1, H2, H3, H4, H6, T1, T2, T3, T4, T6\}$, so

$$P(E) = \frac{n(E)}{n(S)} = \frac{10}{12} = \boxed{\frac{5}{6}}.$$