Math 105 Prelim #1 – September 30, 2003

This exam has 7 problems and 7 numbered pages.

You have 90 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. **Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

Calculators are permitted, but no other aids are allowed.

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name: _____

Instructor: _____

Problem	Score
1	
2	
3	
4	
5	
6	
7	

TOTAL: _____

1. (*8 points*) For each of **(a)** and **(b)**, clearly circle your answer. (*You do not need to show your work, or justify these answers, just circle your choices.*)

Given a set of data points $(x_1, y_1), \ldots, (x_n, y_n)$ we can calculate the least squares line $Y = mx + b$ and the coefficient of correlation r.

(a) If we find that $r = -0.9$, which of the following statements best describes what we can tell about the slope m?

    i. $m = -0.9$

    ii. m is negative, but we do not know the exact value of m

    iii. we cannot determine anything about m

    iv. m is positive, but we do not know the exact value of m

    v. $m = 0.9$

(b) Which statement best describes what this value of r tells us about how the least squares line compares to the data points?

    i. without the equation for the least squares line, we cannot tell anything

    ii. most of the data points are far from the least squares line

    iii. most of the data points are close to the least squares line

2. (*15 points*) Suppose the cost of producing EZ-CHEEZ is linear, that the cost of producing 10 cases is 500 dollars, and that the cost of producing 20 cases is 650 dollars.

(a) Write a linear cost function $C(x)$ that expresses the cost of producing x cases of EZ-CHEEZ.

(b) Suppose that you receive 20 dollars for each case sold. Write a linear revenue function $R(x)$ expressing the revenue earned from the sale of x cases.

(c) Determine how many cases you must produce to break even.

3. (*20 points*) Solve the system of equations

$$
\begin{aligned}
2y + 4z &= 2x \\
4y &= 4 - 6z \\
4x - z &= 5.
\end{aligned}
$$

4. (*10 points*) Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & -2 \\ 3 & -6 \end{bmatrix}.$$

5. (*12 points*) Consider the matrices B and C given by

$$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 2 & 1 \end{bmatrix}.$$

(a) Find BC if it exists. If it does not exist, explain why.

(b) Find CB if it exists. If it does not exist, explain why.

6. (*15 points*) After a genetics experiment on 50 cacti, Prof. G. Mendel recorded the number of cacti that possessed certain characteristics. His observations were as follows.

22 were spiny
25 had flowers
39 produced sweet fruit
9 were spiny and had flowers
20 had flowers and produced sweet fruit
6 had all three traits
4 had none of the traits

(a) Find the number of cacti that were spiny and produced sweet fruit.

(b) How many cacti were spiny and had neither flowers nor produced sweet fruit?

(c) How many cacti were not spiny but had flowers and produced sweet fruit?

7. (*20 points*) Suppose that you flip a fair coin, and then you roll a standard six-sided die.

(a) List the sample space of all possible outcomes of this experiment.

(b) What is the probability that you flip a head and roll a 6?

(c) What is the probability that you flip a tail and roll an even number?

(d) What is the probability that you either flip a tail or roll an even number?

(e) What is the probability that you do not roll a 5?