Math 105.04 Fall 2003
Planes, Lines, and Systems of Equations

The set of all points (x, y, z) where x, y, and z are all real numbers is called *three dimensional Euclidean space*. Sometimes this is denoted $\mathbb{R}^3$, or called 3-space.

We can describe points in 3-space using a Cartesian coordinate system, just as in the plane (two dimensional Euclidean space).

In the plane, the equation $y = mx + b$ describes a line of slope m, and y-intercept b. A line can be written more generally as

$$ax + by = c.$$

Solving for y, we find this line has slope $-a/b$, and y-intercept c/b.

Thus, a line is a one dimensional subset of two dimensional space.

In a similar manner, the equation

$$ax + by + cz = d$$

describes the most general two dimensional subset of three dimensional space, namely a plane.

The plane passes through the points $(d/a, 0, 0)$, $(0, d/b, 0)$, and $(0, 0, d/c)$ as can be easily checked.

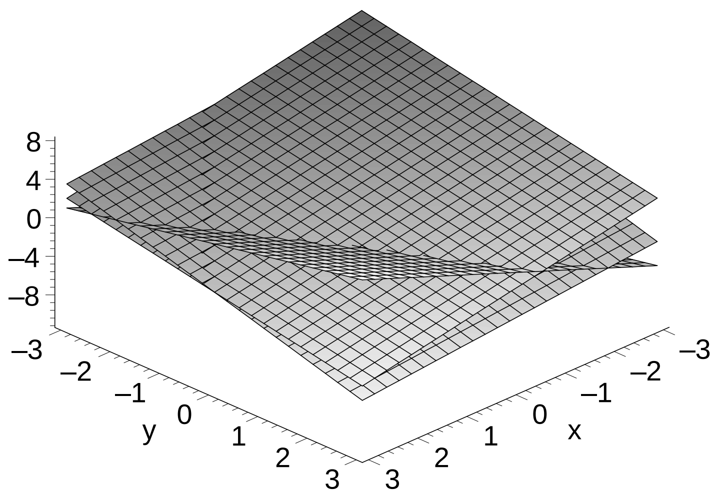
Consider three planes in 3-space. They could intersect

- at no points. The planes could be mutually parallel, one plane may not intersect the other two.
- at a unique point. The planes describing the coordinate x-plane, y-plane, and z-plane intersect at a single point, the origin $(0, 0, 0)$.
- at infinitely many points. In this case, the three planes intersect in a line.

Example: The system

$$\begin{cases} 2x + y - z = 2 \\ x + 3y + 2z = 1 \\ x + y + z = 2 \end{cases}$$

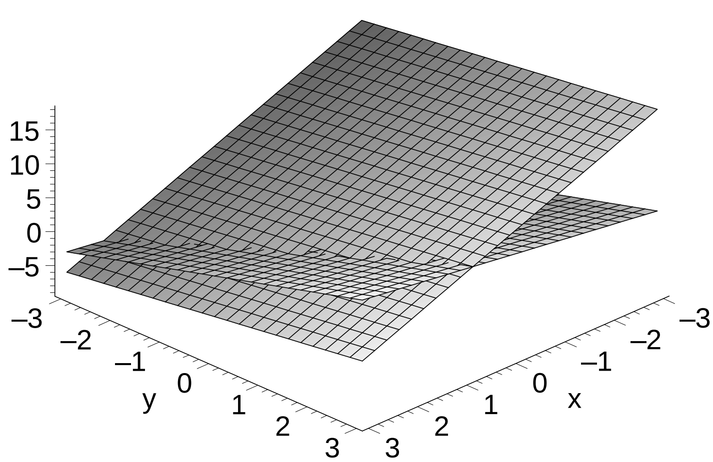
intersects at the point $(2, -1, 1)$.



Example: The system

$$\begin{cases} x + 2y - z = 0 \\ 3x - y + z = 6 \\ -2x - 4y + 2z = 0 \end{cases}$$

intersects along the line given parametrically by $(\frac{15}{7}z + \frac{12}{7}, \frac{4}{7}z - \frac{6}{7}, z)$.



Example: The system

$$\begin{cases} 2x - 2y = -2 \\ y + z = 4 \\ x + z = 1 \end{cases}$$

is inconsistent and has no solutions.