MATH 105 - HOMEWORK 7

Section 8.1

46) There are 10 choices for the first spot, 10 for the second, and so on. By the multiplication principle, there are therefore 10^9 = one billion possible social security numbers. Since there are only 281 million americans, there are enough social security numbers for everybody.

Section 8.2

40a) Since order does not matter, there are $\binom{6}{2} = 15$ ways to pick a good hitter. Similarly, there are $\binom{8}{1} = 8$ ways to pick a poor hitter. By the multiplication principle, there are $15 * 8 = 120$ ways for the coach to pick exactly 2 good hitters and 1 poor hitter.
b) Once again, order does not matter, so the number of ways is $\binom{6}{3} = 20$.
c) The number of ways to choose at least two good hitters is the number of ways to choose exactly 2 good hitters (which is the answer to part a) plus the number of ways to choose exactly 3 good hitters (which is the answer to part b). Therefore, there are $120+20 = 140$ total ways to pick a three players, where at least two of the three are good hitters.

Section 8.3

28) The number of distinguishable permutations of the letters l, i, t, t, l, e is $\frac{6!}{2!2!1!1!} = \frac{6!}{2!2!} = 180$, and the number of distinguishable permutations that spell the word 'little' is clearly only 1. So, the probability is $\frac{1}{180}$.

48) The number of ways of selecting exactly three kings out of a possible four is $\binom{4}{3} = 4$, which is therefore also the number of ways of selecting three aces out of a possible four. There are $52 - 4 - 4 = 44$ cards in the deck that are neither aces nor kings, from which we must select the remaining 7 cards for the hand. So, the total number of bridge hands with exactly three aces and three kings is, by the multiplication principle, $4 * 4 * \binom{44}{7}$. Since the total number of bridge hands is $\binom{52}{13}$, the probability of getting a hand that has exactly three aces and exactly three kings is $\frac{4*4*\binom{44}{7}}{\binom{52}{13}} \simeq .00097$.

Section 8.4

6) The probability of having at least three boys is the probability of having exactly three boys, plus the probability of having exactly four boys, plus the probability of having five boys: $\binom{5}{3}(\frac{1}{2})^3(\frac{1}{2})^2 + \binom{5}{4}(\frac{1}{2})^4(\frac{1}{2})^1 + \binom{5}{5}(\frac{1}{2})^5(\frac{1}{2})^0 = (\frac{1}{2})^5(\binom{5}{3} + \binom{5}{4} + \binom{5}{5}) = \frac{1}{32}(10 + 5 + 1) = \frac{16}{32} = \frac{1}{2}$.

14) The desired probability is equal to the binomial probability of rolling no

1s plus the binomial probability of rolling one 1: $\binom{12}{0}(\frac{1}{6})^0(\frac{5}{6})^{12}+\binom{12}{1}(\frac{1}{6})^1(\frac{5}{6})^{11} \simeq$ .381.

18) The idea behind this problem is the same as in number 6: $\binom{6}{3}(\frac{1}{2})^3(\frac{1}{2})^3 + \binom{6}{4}(\frac{1}{2})^4(\frac{1}{2})^2 + \binom{6}{5}(\frac{1}{2})^5(\frac{1}{2})^1 + \binom{6}{6}(\frac{1}{2})^6(\frac{1}{2})^0 = (\frac{1}{2})^6(\binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6}) = \frac{1}{64}(20 + 15 + 6 + 1) = \frac{42}{64} = \frac{21}{32} \simeq .656$.

40a) This is just another application of binomial probability: $\binom{75}{5}(.05)^5(.95)^{70} \simeq$ .149.
b) $\binom{75}{0}(.05)^0(.95)^{75} = (.95)^{75} \simeq .021$.
c) The complement of the event "at least one item is defective" is the event "no item is defective," which is the answer from part b. Therefore, the desired probability is $1 - (.95)^{75} \simeq .979$.