Math 105 Fall 2003 Solutions Assignment 6[100pts]

8.2 12a[5pts] Selecting a committee of 4 among 30 members is choosing a subset of 4 out of the set of 30 (no repetitions, the order does not matter). Thus there are $\binom{30}{4}$ different ways to select this committee. $\binom{30}{4} = \frac{30\times29\times28\times27}{4\times3\times2\times1} = 5\times29\times7\times27 = 27405$.

12b[5pts] There are 30 ways to select a committee of 1, $\binom{30}{2} = 435$ ways to select a committee of 2, and $\binom{30}{2} = 4060$ ways to select a committee of 3. Thus there are $30 + 435 + 4060 = 4525$ ways to select a committee of at least 1 and at most 3 members.

8.2 20[10pts] There are 12 choices for the child left without a chair. Given any of these choices, there are 11! arrangements of the remaining 11 children on the row of chairs. Thus there are $12\times(11!) = 12!$ different ends to the game if we pay attention to the way kids are seated at the end.

8.2 28[10pts] There are $\binom{7}{3}$ different ways to select 3 secretaries among 7. For any of these choices there are 3! ways to assign the chosen secretaries to the 3 managers. Thus there are $\binom{7}{3}\times 3! = 7\times6\times5 = 210$ different ways to select and assign 3 secretary. In fact, slecting and assigning produces a permutation of 7 taken 3 at a time and $P(7,3) = 7\times6\times5\times = 210$.

8.2 38b[5pts] There are 12 face cards (3 for each suit). Hence there are $\binom{40}{5} = 658,008$ hands containing no face cards.

38e[5pts] There are 13 ways to pick a heart, $\binom{13}{2}$ ways to pick two diamonds and $\binom{13}{2}$ ways to pick two clubs.Hence there are $13\times\binom{13}{2}\times\binom{13}{2} = 79,092$ hands with 1 heart, 2 diamonds and 2 clubs.

8.3 10[10pts] There are 12 face cards and $\binom{12}{2} = 66$ hands of two cards contining only face cards. The total number of 2-card hands is $\binom{52}{2} = 1326$. Hence the probability taht a 2-card hand contains only face cards is $\binom{12}{2}/\binom{52}{2} = 66/1326 \approx .05$

8.3 14[5pts] There are 26 letters in the alphabet. A word is a sequence of 5 letters and there are $(26)^5$ different such sequences. There are $(26)^4$ such sequences that starts with the letter p. Hence the desired probability is $(26)^4/(26)^5 = 1/26 \approx .038$.

8.3 26[10pts] The drawer contains 17 socks. Hence there are $\binom{17}{2} = 136$ different ways to pick a pair. Of those pairs, there are $\binom{9}{2} = 36$ pairs of black socks, $\binom{6}{2=15}$ pairs of brown socks and 1 pair of blue socks. The probability to pick a matching pair is

$$\frac{\binom{9}{2}+\binom{9}{2}+1}{\binom{17}{2}} = \frac{52}{136} = \frac{13}{34} \approx .38.$$

8.3 44[10pts] We can describe picking a hand of two pairs as follows. Pick two values out of 13. For each of these two values, pick two cards out of four with this value. Finally pick a fifth card out of the $44 = 4\times11$ cards whose value is different form those two we first picked. Thus there are $\binom{13}{2}\times\binom{4}{2}\times\binom{4}{2}\times 44 = 13\times6\times6\times6\times44 = 123,552$ such hands.

Another way to do it is to pick three values out of thirteen, then pick one value out of those three (for the single card), pick one card out of four with that value, then pick two out of four for each of the two remaining chosen values. This gives $\binom{13}{3} \times 3 \times 4 \times \binom{4}{2} \times \binom{4}{2} = 13 \times 2 \times 11 \times 3 \times 4 \times 6 \times 6 = 123,552$ hands. As there are $\binom{52}{5} = 2,598,960$ different 5-card hands, the desired probability is $123,552/2,598,960 \approx .05$.

8.3 50a[5pts] There are $\binom{21}{6} = 54,264$ ways to pick 6 books out of $9 + 5 + 7 = 21$. There are $\binom{9}{3} = 84$ ways to pick three books by Hughes, $\binom{7}{3} = 35$ ways to pick three books by Morrison. Thus the probability that the 6 books picked are 3 Hughes and 3 Morrison is $\binom{9}{3} \times \binom{7}{3}/\binom{21}{6} \approx .054$.

50d[5pts] There are $\binom{9}{6} = 84$ ways to pick 6 Hughes books, $\binom{9}{5} \times 12 = 1512$ ways to pick 5 Hughes books and another book that is not by Hughes, $\binom{9}{4} \times \binom{12}{2} = 8316$ ways to pick 4 Hughes book and 2 others not by Hughes. So the total number of ways to choose at least 4 Hughes book is $84 + 1512 + 8316 = 9912$. The probability that this happens is $9912/54,264 \approx .18$.

50f[5pts] The number of ways to pick no books by Baldwin is $\binom{16}{6} = 8,008$. The number of ways to pick 1 book by Baldwin is $5 \times \binom{16}{5} = 21,840$. The number of ways to pick two books by Baldwin is $\binom{5}{2} \times \binom{16}{4} = 18,200$. Hence there are $8,008 + 21,840 + 18,200 = 48,048$ ways to pick no more than 2 books by Baldwin. The probability of that happening is $48,048/54,264 \approx .89$.

8.3 52[10pts] There are $\binom{99}{6} = 1,120,529,256$ different picks. To have 5 correct numbers, pick one of the six correct numbers and replace it by one of the 93 remaining numbers. Thus there are $6 \times 93 = 558$ different picks that have 5 correct numbers. The probability to get exactly 5 correct numbers is $558/\binom{99}{6} \approx 5 \times 10^{-7}$.