Math 105 Fall 2003 Homework 5 Solutions [100 points]

Section 7.6

#6 [10 pts]. We have $P(R_1'|Q) = 1 - P(R_1|Q)$, and so by Bayes' Theorem,

$$
\begin{aligned}
P(R_1'|Q) &= 1 - \frac{P(R_1)\cdot P(Q|R_1)}{P(R_1)\cdot P(Q|R_1) + P(R_2)\cdot P(Q|R_2) + P(R_3)\cdot P(Q|R_3)} \\
&= 1 - \frac{(.40)(.05)}{(.40)(.05) + (.30)(.60) + (.60)(.35)} \\
&= 1 - \frac{.02}{.02 + .18 + .21} \\
&= 1 - \frac{.02}{.41} = \frac{.39}{.41} \cong \boxed{0.95} = 95\%.
\end{aligned}
$$

#12 [10 pts]. Let A = from Supplier A, B = from Supplier B, and D = damaged bags of cement. Then by Bayes' Theorem,

$$
\begin{aligned}
P(A|D) &= \frac{P(A)\cdot P(D|A)}{P(A)\cdot P(D|A) + P(B)\cdot P(D|B)} \\
&= \frac{(.70)(1-.90)}{(.70)(1-.90) + (.30)(1-.95)} \\
&= \frac{.07}{.07 + .015} = \frac{.07}{.085} \cong \boxed{0.82} = 82\%.
\end{aligned}
$$

#16 [10 pts]. Let D = defective appliances. Then by Bayes' Theorem,

$$
\begin{aligned}
P(A|D) &= \frac{P(A)\cdot P(D|A)}{P(A)\cdot P(D|A) + P(B)\cdot P(D|B) + P(C)\cdot P(D|C)} \\
&= \frac{(.15)(.01)}{(.15)(.01) + (.40)(.015) + (.45)(.02)} \\
&= \frac{.0015}{.0015 + .006 + .009} = \frac{.0015}{.0165} \cong \boxed{0.09} = 9\%.
\end{aligned}
$$

#26 [10 pts]. Let V = has the HIV virus, and let T = tested positive. Note that the probability of a false positive is $P(T|V') = 2\%$ and the probability of a false negative is $P(T'|V) = 5\%$. Also, note that $P(V) = \frac{780{,}000}{295 \text{ million}} \cong 0.0026$. We want $P(V|T)$, and by Bayes' Theorem we have:

$$\begin{aligned} P(V|T) &= \frac{P(V)\cdot P(T|V)}{P(V)\cdot P(T|V) + P(V')\cdot P(T|V')} \\ &= \frac{(.0026)(1-.05)}{(.0026)(1-.05)+(1-.0026)(.02)} \\ &= \frac{.00247}{.00247+.019948} = \frac{.00247}{0.022418} \cong \boxed{0.11} = 11\%. \end{aligned}$$

#32 [10 pts]. Let S = was wearing a seat belt, and let K = were killed. Then by Bayes' Theorem,

$$\begin{aligned} P(S|K) &= \frac{P(S)\cdot P(K|S)}{P(S)\cdot P(K|S) + P(S')\cdot P(K|S')} \\ &= \frac{(.49)(.27)}{(.49)(.27)+(.51)(.50)} \\ &= \frac{.1323}{.1323+.255} = \frac{.1323}{0.3873} \cong \boxed{0.34} = 34\%. \end{aligned}$$

<u>Section 8.1</u>

#14 [10 pts]. Using the multiplication principle, the number of meals is $3\cdot 8\cdot 5 = \boxed{120}$.

#20 (a) [6 pts]. $\frac{7!}{3!1!1!1!1!} = 7\cdot 6\cdot 5\cdot 4 = \boxed{840}$.

(b) [6 pts]. $\frac{6!}{2!2!1!1!} = \frac{720}{4} = \boxed{180}$.

(c) [6 pts]. $\frac{7!}{3!2!1!1!} = \frac{7\cdot 6\cdot 5\cdot 4}{2\cdot 1} = \boxed{420}$.

#36 [10 pts]. $P(20,9) = \frac{20!}{(20-9)!} = \frac{20!}{11!} = 20\cdot 19\cdot 18\cdot 17\cdot 16\cdot 15\cdot 14\cdot 13\cdot 12 = \boxed{60949324800}$.

#52 (a) [6 pts]. We use the multiplication principle, noting that since we are looking at completed circuits, the starting location doesn't matter. We then have 9 other cities to choose from, then 8, etc., down to 1. So there are $9! = \boxed{362880}$ different circuits.

(b) [6 pts]. Since in our calculations in (a) we travelled every circuit forward and backward, we only need to check half of the number above. So we only need to check $\frac{9!}{2} = \boxed{181440}$ circuits.