Review page 117-122 #6 [6 pts] Begin with the system

$$\begin{cases} x - y = 3 \\ 2x + 3y + z = 13 \\ 3x - 2z = 21 \end{cases}$$

and perform

$$-2R_1 + R_2 \mapsto R_2 \;\text{ and }\; -3R_1 + R_3 \mapsto R_3$$

to yield

$$\begin{cases} x - y = 3 \\ 5y + z = 7 \\ 3y - 2z = 12 \end{cases}$$

followed by

$$-3R_2 + 5R_3 \mapsto R_3$$

to find

$$\begin{cases} x - y = 3 \\ 5y + z = 7 \\ -13z = 39. \end{cases}$$

Thus, from R_3 we have that $z = -3$. Back substituting gives, from R_2, that $5y = 7 - z = 7 + 3$ or $y = 2$, and from R_1 that $x = 3 + y = 3 + 2 = 5$. Hence, the unique solution is

$$(x, y, z) = (5, 2, -3).$$

Review page 117-122 #10 [6 pts] The augmented matrix associated with the system is

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 3 & 2 & 0 & 8 \\ -1 & 0 & 2 & 10 \end{array}\right]$$

Performing the row operations $-3R_1 + R_2 \mapsto R_2$ and $R_1 + R_3 \mapsto R_3$ produces

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 0 & 2 & 6 & -7 \\ 0 & 0 & 0 & 15 \end{array}\right]$$

From the last row we see that $0 = 15$, which is obviously false, so that we conclude the system is *inconsistent* and there is no solution.

Review page 117-122 #36 [6 pts] Let

$$A = \begin{bmatrix} 1 & 3 & 6 \\ 4 & 0 & 9 \\ 5 & 15 & 30 \end{bmatrix}.$$

To find A^{-1} we form the augmented system $[A|I]$ and attempt to row reduce it to $[I|A^{-1}]$. Thus,

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 6 & 1 & 0 & 0 \\ 4 & 0 & 9 & 0 & 1 & 0 \\ 5 & 15 & 30 & 0 & 0 & 1 \end{array}\right].$$

The row operations $-4R_1 + R_2 \mapsto R_2$, $-5R_1 + R_3 \mapsto R_3$ produce

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 6 & 1 & 0 & 0 \\ 0 & -12 & -15 & -4 & 1 & 0 \\ 0 & 0 & 0 & -5 & 0 & 1 \end{array}\right].$$

From the third row of all zeroes, we see that the matrix is *singular*. That is, no inverse exists.

Review page 117-122 #52 [8 pts]

Let x be Tulsa's number of gallons, let y be New Orlean's number of gallons, and let z be Ardmore's number of gallons. The system may then be written as:

$$\begin{cases} .5x + .4y + .3z = 219000 \text{ (Chicago)} \\ .2x + .4y + .4z = 192000 \text{ (Dallas)} \\ .3x + .2y + .3z = 144000 \text{ (Atlanta)}. \end{cases}$$

We form the augmented system

$$\left[\begin{array}{ccc|c} 0.5 & 0.4 & 0.3 & 219000 \\ 0.2 & 0.4 & 0.4 & 192000 \\ 0.3 & 0.2 & 0.3 & 144000 \end{array}\right].$$

The row operations $2R_1 - 5R_2 \mapsto R_2$, $3R_1 - 5R_3 \mapsto R_3$ produce

$$\left[\begin{array}{ccc|c} 0.5 & 0.4 & 0.3 & 219000 \\ 0 & -1.2 & -1.4 & -522000 \\ 0 & 0.2 & -0.6 & -63000 \end{array}\right],$$

and the row operations $-2R_3 + R_1 \mapsto R_1$, $R_2 + 6R_3 \mapsto R_3$ produce

$$\left[\begin{array}{ccc|c} 0.5 & 0 & 1.5 & 345000 \\ 0 & -1.2 & -1.4 & -522000 \\ 0 & 0 & -5 & -900000 \end{array}\right].$$

Next, the row operations $0.3R_3 + R_1 \mapsto R_1$, $-14R_3 + 50R_2 \mapsto R_2$ produce

$$\left[\begin{array}{ccc|c} 0.5 & 0 & 0 & 75000 \\ 0 & -60 & 0 & 225000 \\ 0 & 0 & -5 & 180000 \end{array}\right].$$

Finally, multiplying R_1 by 2, multiplying R_2 by $-1/60$, and multiplying R_3 by $-1/5$, gives that 150000 gallons were produced at Tulsa, 225000 gallons were produced at New Orleans, and

180000 gallons were produced at Ardmore.

7.1 #36 [4 pts] $Y' = U \setminus Y = \{2, 3, 4, 5, 7, 9\} \setminus \{3, 5, 7, 9\} = \{2, 4\}$

7.1 #40 [6 pts] Since $X' = U \setminus X = \{2, 3, 4, 5, 7, 9\} \setminus \{2, 3, 4, 5, \} = \{7, 9\}$, and $Y' = \{2, 4\}$ (from #36) we have

$$X' \cap (Y' \cup Z) = \{7, 9\} \cap (\{2, 4\} \cup \{2, 4, 5, 7, 9\}) = \{7, 9\} \cap \{2, 4, 5, 7, 9\} = \{7, 9\}.$$

7.1 #54 [2 pts each] (a) TRUE, a is a member of the set B

(b) FALSE, while the elements b and d are members of B, the element d is NOT a member of B. The element $\{d\}$ however IS a member of B. Note that $d \neq \{d\}$.

(c) TRUE, the element $\{d\}$ is a member of B.

(d) FALSE, while the element $\{d\}$ is a member of B, which is written $\{d\} \in B$, the set consisting of the element d is not a subset of B since the element d is not a member of B. It would be true to write $\{\{d\}\} \subseteq B$.

(e) TRUE, the element $\{e, f\}$ is a member of B.

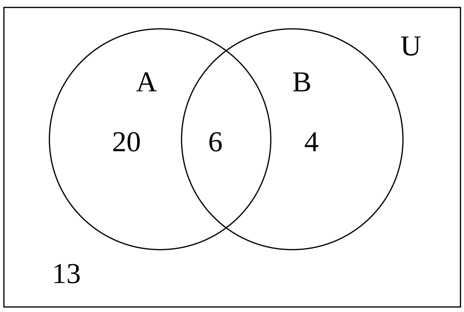
(f) TRUE, the set $\{a, \{e, f\}\}$ consisting of the elements a and $\{e, f\}$ is a proper subset of B since each element of the set $\{a, \{e, f\}\}$ is also an element of B, and there are elements of B that are not in $\{a, \{e, f\}\}$, namely a, b, c, and $\{d\}$.

(g) FALSE, as noted in (e) the element $\{e, f\}$ is a member of B, and similar to (d) the set consisting of the elements e and f is not a subset of B since neither the element e nor the element f are members of B. It would be true to write $\{\{e, f\}\} \subset B$.

7.2 #20 [4 pts] By the union rule for sets, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, so if $n(A) = 12$, $n(B) = 27$, and $n(A \cup B) = 30$, then $30 = 12 + 27 - n(A \cap B)$ so that

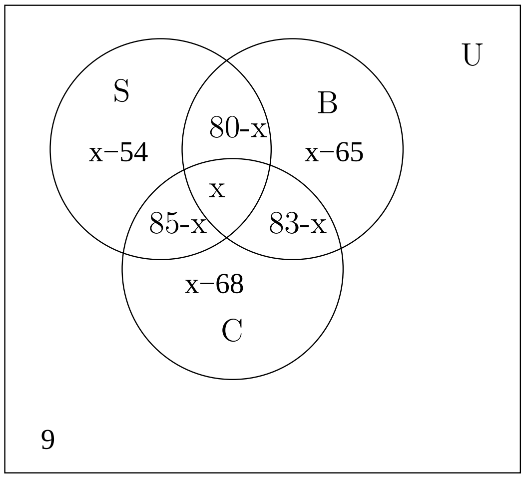
$$n(A \cap B) = 9.$$

7.2 #24 [6 pts]



Since $n(A) = 26$, $n(B) = 10$, $n(A \cup B) = 30$, and $n(A') = 17$, we find that $n(A \cap B) = n(A) + n(B) - n(A \cup B) = 26 + 10 - 30 = 6$. Since $n(A') = 17$ and $n(B \cap A') = 4$, we see that $n((A \cup B)') = 13$.

7.2 #40 [10 pts] Let $S = \{$set of surveyed mathematics professors who invest their hard-earned money in stocks$\}$, $B = \{\cdots$ in bonds$\}$ and $C = \{\cdots$ in certificates of deposits$\}$. Then $n(S) = 111$, $n(B) = 98$, $n(C) = 100$, $n(S \cap B) = 80$, $n(B \cap C) = 83$, $n(S \cap C) = 85$, and $n((S \cup B \cup C)') = 9$. We seek $n(S \cap B \cap C)$. Let $x = n(S \cap B \cap C)$. Now we can fill in the Venn diagram:



Since the sum of all 3 regions is 141 (that is, 150-9), we see that

$$141 = (x - 54) + (80 - x) + (x - 65) + x + (85 - x) + (83 - x) + (x - 68) = 61 + x.$$

Thus, $x = 80$.

7.3 #16 [8 pts] The sample space is $S = \{RRR, RRW, RWR, RWW, WRR, WRW, WWR, WWW\}$ where we have written R to indicate that the student's guess was correct, and W to indicate that his guess was incorrect, with each outcome equally likely.

(a) The event that the student gets three answers wrong is $A = \{WWW\}$. Thus,

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}.$$

(b) The event that the student gets two answers correct is $B = \{WRR, RWR, RRW\}$. Thus,

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{8}.$$

(c) The event that the student gets only the first answer correct is $C = \{RWW\}$. Thus,

$$P(C) = \frac{n(C)}{n(S)} = \frac{1}{8}.$$

7.3 #34 [6 pts] Let S be the sample space consisting of all 52 different cards in a standard deck, and let E be the event that the drawn card is either a heart or a spade. Since there are 13 hearts, and 13 spades, and no overlap between the suits, the probability in question is

$$P(E) = \frac{26}{52} = \frac{1}{2}.$$

Alternatively, if we let $\heartsuit$ be the event a heart is drawn, and $\spadesuit$ be the event that a spade is drawn, then

$$P(E) = P(\heartsuit) + P(\spadesuit) - P(\heartsuit \cap \spadesuit) = \frac{13}{52} + \frac{13}{52} - \frac{0}{52} = \frac{1}{2}.$$

7.3 #40 [6 pts] In order to use the "Basic Probability Principle" on page 329, we must have a sample space whose elements are each equally likely. That is achieved by letting $S = \{W, W, O, O, O, Y, Y, Y, Y, Y, B, B, B, B, B, B, B, B\}$ where we use W for white, O for orange, Y for yellow, and B for black, and also do not distinguish between the marbles. Thus, if F is the event that an orange marble is drawn, and G is the event that a yellow marble is drawn, then $E = F \cup G$ is the event that either an orange or a yellow marble is drawn. Hence,

$$P(E) = P(F \cup G) = P(F) + P(G) - P(F \cap G) = \frac{3}{18} + \frac{5}{18} - 0 = \frac{8}{18} = \frac{4}{9}.$$

Note it is incorrect to use Y for BOTH the elements in the sample space, and the event that a yellow is drawn.

7.4 #12 [4 pts] Let S be the set of 36 possible pairs of outcomes when two dice are rolled. The event that the first die is a 3 is $A = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\}$ and the event that the sum is 8 is $B = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$. Note that $A \cap B = \{(3,5)\}$. Thus, the event that either the first die is 3 or the sum is 8 is $A \cup B$. This occurs with probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{6}{36} + \frac{5}{36} - \frac{1}{36} = \frac{10}{36}.$$

7.4 #18 [2 pts each] Let $S = \{$Mom, Aunt, Aunt, Uncle, Uncle, Uncle, Brother, Brother, Male Cousin, Female Cousin$\}$. Let B be the event that a brother arrives first, let U be the event that an uncle arrives first, let M be the event that her mother arrives first, and let C be the event that a cousin arrives first.

(a) Since no brothers are uncles,

$$P(U \cup B) = P(U) + P(B) = \frac{3}{10} + \frac{2}{10} = \frac{1}{2}.$$

(b) Since no brothers are cousins,

$$P(B \cup C) = P(B) + P(C) = \frac{2}{10} + \frac{2}{10} = \frac{2}{5}.$$

(c) Since no brothers are mothers,

$$P(B \cup M) = P(B) + P(M) = \frac{2}{10} + \frac{1}{10} = \frac{3}{10}.$$