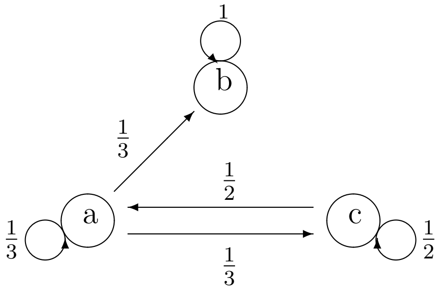
10.1 14[5pts] Yes, it is a transition matrix.



10.1 22[10pts]

$$D = \begin{pmatrix} .3 & .2 & .5 \\ 0 & 0 & 1 \\ .6 & .1 & .3 \end{pmatrix}, \ D^2 = \begin{pmatrix} .39 & .11 & .5 \\ .6 & .1 & .3 \\ .36 & .15 & .49 \end{pmatrix}, \ D^3 = \begin{pmatrix} .417 & .128 & .455 \\ .36 & .15 & .49 \\ .402 & .121 & .477 \end{pmatrix}$$

The probability of a transition from 1 to 2 after 3 repetitions is .128.

10.1 30[10pts]

$$P = \begin{pmatrix} .85 & .10 & .05 \\ 0 & .80 & .20 \\ 0 & 0 & .1 \end{pmatrix}.$$

The probability of transition from G1 to G2 must be .20 because the row must add up to 1.

10.1 38 (a)[10pts] Using L,C, I for liberal, conservative and independent the transition matrix P is given by

	L	C	I
L	.80	.15	.05
C	.20	.70	.10
I	.20	.20	.60

b)[5pts] The initial distribution is $X_0 = (.40, .45, .15)$.

c-f)[10pts] One month later in July, the distribution is $X_1 = X_0P = (.44, .405, .155)$. Two month later in August $X_2 = X_1P = (.464, .3805, .1555)$. In September: $X_3 = X_2P = (.4784, .36705, .15455)$. In October: $X_4 = X_3P = (.48704, .36505, .153355)$. (Simplifications to two decimals are acceptable).

10.2 8[10pts] Let $V = v_1, v_2)$ and solve $VP = V$ with $v_1 + v_2 = 1$. This gives the system

$$\begin{cases} \frac{2}{3}v_1 + \frac{1}{8}v_2 = v_1 \\ \frac{1}{3}v_1 + \frac{7}{8}v_2 = v_2 \\ v_1 + v_2 = 1 \end{cases}$$

Putting the unknowns v_1, v_2 on the left hand side gives

$$\begin{cases} -\frac{1}{3}v_1 & + & \frac{1}{8}v_2 & = & 0 \\ \frac{1}{3}v_1 & - & \frac{1}{8}v_2 & = & 0 \\ v_1 & + & v_2 & = & 1 \end{cases}$$

The second row is euqal to minus the first so the system is equivalent to

$$\begin{cases} -\frac{1}{3}v_1 & + & \frac{1}{8}v_2 & = & 0 \\ v_1 & + & v_2 & = & 1 \end{cases}$$

Multiplying the fisrt row by 3 and adding to the second row gives $\frac{11}{8}v_2 = 1$, that is $v_2 = 8/11$. Using the second row, this gives $v_1 = 1 - 8/11 = 3/11$. The equilibrium vector is $V = (3/11, 8/11)$.

10.2 14[20pts] The equation $VP = V$, $V = (v_1, v_2, v_3)$ with $v_1 + v_2 + v_3) = 0$ gives the system

$$\begin{cases} v_1 & + & v_2 & + & v_3 & = & 1 \\ .16v_1 & + & .43v_2 & + & .86v_3 & = & v_1 \\ .28v_1 & + & .12v_2 & + & .05v_3 & = & v_2 \\ .56v_1 & + & .45v_2 & + & .09v_3 & = & v_3 \end{cases}$$

which gives

$$\begin{cases} v_1 & + & v_2 & + & v_3 & = & 1 \\ -.84v_1 & + & .43v_2 & + & .86v_3 & = & 0 \\ .28v_1 & - & .88v_2 & + & .05v_3 & = & 0 \\ .56v_1 & + & .45v_2 & - & .91v_3 & = & 0 \end{cases}$$

Solving gives the equilibrium vector $V = (\frac{7783}{16{,}799}, \frac{2828}{16{,}799}, \frac{6188}{16{,}799})$.

10.2 26[10pts] The quilibrium vector is the solution $V = (v_1, v_2)$ of $VP = V$ with $v_1 + v_2 = 1$, that is

$$\begin{cases} .95v_1 & + & .80v_2 & = & v_1 \\ .05v_1 & + & .20v_2 & = & v_2 \\ v_1 & + & v_2 & = & 1 \end{cases}.$$

This becomes

$$\begin{cases} -.05v_1 & + & .80v_2 & = & 0 \\ .05v_1 & - & .80v_2 & = & 0 \\ v_1 & + & v_2 & = & 1 \end{cases}.$$

The first row is minus the second row and the system reduces to

$$\begin{cases} -.05v_1 & + & .80v_2 & = & 0 \\ v_1 & + & v_2 & = & 1 \end{cases}$$

The solution is $V = (16/17, 1/17)$.

10.2 38[10pts] The quilibrium vector is the solution $V = (v_1, v_2)$ of $VP = V$ with $v_1 + v_2 = 1$ with P given in the book. Hence

$$\begin{cases} .12v_1 & + & .54v_2 & = & v_1 \\ .88v_1 & + & .46v_2 & = & v_2 \\ v_1 & + & v_2 & = & 1 \end{cases}.$$

This becomes

$$\begin{cases} -.88v_1 & + & .54v_2 & = & 0 \\ .88v_1 & - & .54v_2 & = & 0 \\ v_1 & + & v_2 & = & 1 \end{cases}.$$

The first row is minus the second row and the system reduces to

$$\begin{cases} -.88v_1 & + & .54v_2 & = & 0 \\ v_1 & + & v_2 & = & 1 \end{cases}$$

The solution is $V = (27/71, 44/71)$. Since $27/71 \approx .38$, about 38% of the letters in English text are expected to be vowels.