UNIVERSITY OF REGINA
Department of Mathematics & Statistics
Colloquium

Speaker: Michael Kozdron

Date: Friday, November 18, 2005
Time: 3:30 p.m.
Location: College West 307.18 (Math & Stats Lounge)

Title: *An Introduction to the Schramm-Loewner Evolution*

Abstract: The interplay between probability and complex analysis was really first exploited by Paul Lévy in the 1950's who realized that two-dimensional Brownian motion was conformally invariant. Since it had been proved earlier by Monroe Donsker that the scaling limit of simple random walk is Brownian motion, we now had an example of a *conformally invariant scaling limit*.

For the past several decades physicists and mathematicians have studied two-dimensional discrete models with the hope of explaining some of the macroscopic properties of the associated physical system. The Ising model of spin systems is such an example. One approach to the analysis is to determine a scaling limit which is, hopefully, conformally invariant. The so-called holy grail of this program is the self-avoiding walk, a model of polymer chains introduced in the 1940's by the Nobel-prize winning physicist Paul Flory.

Recently, Oded Schramm combined an old equation of Charles Loewner's in complex analysis with some randomness to create a process he called the *stochastic Loewner evolution (SLE)*. The Schramm-Loewner evolution (as it is now named) has led to a flurry of deep results enhancing our understanding of two-dimensional lattice models at criticality. In addition to having settled a long-standing conjecture of Benoît Mandelbrot regarding the Hausdorff (fractal) dimension of the frontier of the Brownian island, thanks to SLE we now have a much better understanding of the scaling limits of percolation, the harmonic exploration process, and the loop-erased random walk. We even have a plausible candidate for the scaling limit of the self-avoiding walk.

Our main focus will be on explaining both the classical Loewner equation, and the Schramm-Loewner evolution. We will discuss many of the properties of SLE which make it so useful, and we will discuss the conjecture of Mandelbrot. This primarily expository talk might also be viewed as providing some background material for Schramm's plenary lecture at the upcoming CMS Winter Meetings in Victoria.