Sociology 405/805
January 20, 2004

Gamma (γ), Tau-a (τ_a), Tau-b (τ_b), Tau-c (τ_c), and Somer's d

These notes give formulae and an example calculation of measures of association for a cross-classification of two variables, where each variable has an ordinal or higher level scale of measurement.

These measures of association are discussed on pages 342-364 of *Statistics: A Tool for the Social Sciences*, third edition, by Lyman Ott, Richard F. Larson, and William Mendenhall. The notation in these notes is that of Ott, Larson, and Mendenhall. The formula for τ_c is taken from the section "Ordinal Association" in G. David Garson, *Statnotes: An Online Textbook*. Another good source of information on these measures is Albert M. Liebetrau, *Measures of Association*, Sage Quantitative Applications in the Social Sciences, number 32.

Notation

N_s is the number of concordant pairs.

N_r is the number of discordant pairs.

n is the sample size.

T, the total number of possible pairs, is $T = n(n-1)/2$.

T_I is the number of pairs with tied ranks on the independent variable.

T_D is the number of pairs with tied ranks on the dependent variable.

m is the minimum of the number of rows or columns (from Garson).

Measures of association

Using the above notation, the formulae for the measures of association are as follows:

$$\gamma = \frac{N_s - N_r}{N_s + N_r}$$

$$\tau_a = \frac{N_s - N_r}{T}$$

$$\text{Somer's } d = \frac{N_s - N_r}{N_s + N_r + T_D}$$

$$\tau_b = \frac{N_s - N_r}{\sqrt{(N_s + N_r + T_D)(N_s + N_r + T_I)}}$$

$$\tau_c = \frac{(N_s - N_r)2m}{n^2(m-1)}$$

Table 1: Cross Classification of $V5$ with $V6$

Response to $V5$	Response on $V6$ Disagree	Neutral	Agree	Total
Disagree	34	24	15	73
Neutral	42	74	67	183
Agree	28	111	292	431
Total	104	209	374	687

Example – Relation of $V5$ and $V6$

The example used is the cross-classification of Table 1, from data in question 15 of the *ssae98.sav* data set. The variables in this question are ordinal scale variables, measuring attitudes on a five-point scale from strongly disagree to strongly agree with the statement. This example provides statistics describing the relationship between attitudes on *V5* (taxes on big corporations should be increased) and *V6* (governments are more interested in helping big business than in helping Canadian citizens). In order to reduce

the number of calculations, the five-point scale has been collapsed to a three-point scale by merging responses 1 and 2 into "disagree" and 4 and 5 into "agree." Response 3 is left unchanged and is termed "neutral."

N_s – Number of concordant pairs

Beginning at the top left of the table and proceeding to the right and down, the number of concordant pairs is

$$\begin{aligned} 34(74+67+111+92) &= 18,496 \\ 24(67+92) &= 8,616 \\ 42(111+292) &= 16,926 \\ 74(292) &= 21,608 \\ N_s &= 65,646 \end{aligned}$$

N_r – Number of discordant pairs

Beginning at the top right of the table and proceeding to the left and down, the number of discordant pairs is

$$\begin{aligned} 15(74+42+111+28) &= 3,825 \\ 24(42+28) &= 1,680 \\ 67(111+28) &= 9,313 \\ 74(28) &= 2,072 \\ N_r &= 16,890 \end{aligned}$$

T – Total number of pairs

$$T = \frac{n(n-1)}{2} = \frac{687 \times 686}{2} = 235,641$$

T_D – Number of ties on dependent variable $V6$

Beginning at the top left of the table and proceeding to the right and down, the number of ties on $V6$ is

$$\begin{aligned} 34(42+28) &= 2,380 \\ 24(74+111) &= 4,440 \\ 15(67+292) &= 5,385 \\ 42(28) &= 1,176 \\ 74(111) &= 8,214 \\ 67(292) &= 19,564 \\ T_D &= 41,159 \end{aligned}$$

T_I – Number of ties on independent variable $V5$

Beginning at the top left of the table and proceeding to the right and down, the number of ties on $V5$ is

$$\begin{aligned} 34(24+15) &= 1,326 \\ 42(74+67) &= 5,922 \\ 28(111+292) &= 11,284 \\ 24(15) &= 360 \\ 74(67) &= 4,958 \\ 111(292) &= 32,412 \\ T_I &= 56,262 \end{aligned}$$

Calculation of measures of association

γ **(Gamma)**

$$\begin{aligned}\gamma &= \frac{N_s - N_r}{N_s + N_r} \\ &= \frac{65,646 - 16,890}{65,646 + 16,890} \\ &= \frac{48,756}{82,536} \\ &= 0.591\end{aligned}$$

τ_a **(Tau-a)**

$$\begin{aligned}\tau_a &= \frac{N_s - N_r}{T} \\ &= \frac{65,646 - 16,890}{235,641} \\ &= \frac{48,756}{235,641} \\ &= 0.207\end{aligned}$$

Somer's d **with** $V6$ **dependent**

$$\begin{aligned}d &= \frac{N_s - N_r}{N_s + N_r + T_D} \\ &= \frac{65,646 - 16,890}{82,536 + 41,159} \\ &= \frac{48,756}{123,695} \\ &= 0.394\end{aligned}$$

Somer's d with $V5$ dependent

$$\begin{aligned} d &= \frac{N_s - N_r}{N_s + N_r + T_I} \\ &= \frac{65,646 - 16,890}{82,536 + 56,262} \\ &= \frac{48,756}{138,798} \\ &= 0.351 \end{aligned}$$

τ_b **(Tau-b)**

$$\begin{aligned} \tau_b &= \frac{N_s - N_r}{\sqrt{(N_s + N_r + T_D)(N_s + N_r + T_I)}} \\ &= \frac{65,646 - 16,890}{\sqrt{(65,646 + 16,890 + 41,159)(65,646 + 16,890 + 56,262)}} \\ &= \frac{48,756}{\sqrt{123,695 \times 138,798}} \\ &= \frac{48,756}{131,029.05} \\ &= 0.372 \end{aligned}$$

τ_c **(Tau-c)**

$$\begin{aligned} \tau_c &= \frac{(N_s - N_r)2m}{n^2(m-1)} \\ &= \frac{(65,646 - 16,890) \times 6}{687^2 \times 2} \\ &= \frac{48,756 \times 6}{471,969 \times 2} \\ &= \frac{292,536}{943,938} \\ &= 0.310 \end{aligned}$$

Last edited January 23, 2004.