# MATH 849 - Topics in Topology
# Stable homotopy theory

Fall 2023

## General information


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| **Instructor:** | Martin Frankland |
| **Email:** | Martin.Frankland@uregina.ca |
| **Office:** | CW 307.17 |
| **Office hours:** | TBA |
| **Format:** | Directed reading along with regular meetings. There are no lectures. |
| **Prerequisite:** | MATH 442 or similar course in algebraic topology. |


## Course outline

Stable homotopy theory is the branch of algebraic topology concerned with phenomena that persist after suspending many times. Its roots lie in the Freudenthal suspension theorem, which guarantees that homotopy groups stabilize after a certain number of suspensions. Unstable homotopy groups do not form a homology theory, but stable homotopy groups do. Fiber sequences and cofiber sequences are different unstably, but they become equivalent stably. The wedge and product of pointed spaces are different unstably, but they become equivalent stably. Unstable cohomology operations can have any arity, but stable cohomology operations are all unary. Those various simplifications provide the stable homotopy category with a more algebraic flavor that is amenable to computations.

The course covers some foundations of stable homotopy theory:

- Freudenthal suspension theorem and stable homotopy groups.
- Spectra and the stable homotopy category.
- Examples: suspension spectra, Eilenberg–MacLane spectra, $K$-theory, Thom spectra.
- Triangulated structure.
- Generalized cohomology theories.
- The Steenrod algebra.

Afterwards, the course will explore some selected advanced topics. Here are some possible topics:

- The Adams spectral sequence.
- Model categories of spectra.
- Bousfield localization of spectra.
- Structured ring spectra.
- Module spectra and the stable Dold–Kan correspondence.
- Complex cobordism and formal group laws.
- Chromatic homotopy theory.

# Textbook

While the course has no official textbook, the primary references are:

- F. Adams, *Stable homotopy and generalised homology.*
- D. Barnes and C. Roitzheim; *Foundations of stable homotopy theory.*

Here are some recommended additional references in stable homotopy theory:

- H.R. Margolis, *Spectra and the Steenrod algebra.*
- M. Hovey, J. Palmieri, and N. Strickland; *Axiomatic stable homotopy theory.*
- A.D. Elmendorf, I. Kriz, M.A. Mandell, and J.P. May; *Rings, modules, and algebras in stable homotopy theory.*
- D. Ravenel, *Nilpotence and periodicity in stable homotopy theory.*
- D. Ravenel, *Complex cobordism and stable homotopy groups of spheres.*

And some useful references in algebraic topology:

- A. Hatcher, *Algebraic Topology.*
- A. Hatcher, *Spectral Sequences in Algebraic Topology.*
- J.P. May, *A Concise Course in Algebraic Topology.*
- R. Switzer, *Algebraic topology—homotopy and homology.*

## Grading scheme

- Term paper: 70%
- Oral presentation: 20%
- Drafts and progress updates: 10%

## Term paper

The main component of the course is to write an expository paper on a selected topic, including some background material and a bibliography. Evaluation will be based on accuracy, quality of the exposition, and contribution beyond the literature (in the form of worked examples or filling in details of proofs).

## Oral presentation

There will be an oral presentation towards the end of the semester. Evaluation will be based on the presentation itself as well as preparation work.

## Missed course work

Information about missed course work can be found in the *Academic Regulations*, section "Deferral of Final Exams or Course Work", available at:

https://www.uregina.ca/student/registrar/resources-for-students/academic-calendars-and-schedule/undergraduate-calendar/sections.html

See in particular the sections "Grounds for Deferral" and "Supporting Documentation".

## Academic integrity

Working with your peers is allowed, in fact encouraged. However, each student must write **their own** work.

Handing in any material copied from the internet or another source is considered cheating. **Cite sources** that you consult, for instance Wikipedia, Math Stack Exchange, or online course notes.

Scholastic offences are taken seriously and will not be tolerated. For more information, please consult the *Student Code of Conduct and Right to Appeal*, section "Academic Misconduct", available at:

https://www.uregina.ca/student/registrar/resources-for-students/academic-calendars-and-schedule/undergraduate-calendar/sections.html

# Accessibility

Any student with special needs who may need accommodation should contact the Centre for Student Accessibility at:

`https://www.uregina.ca/student/accessibility/`

After I receive the letter from the Centre for Student Accessibility, please contact me to discuss the accommodation.