Spooky topology and geometry

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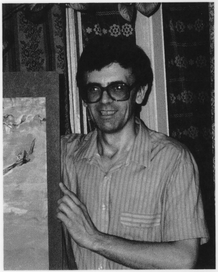
Pi Day
University of Regina
March 13, 2020

Outline

The artwork of Anatoly Fomenko

References:

- http://chronologia.org/en/math_impressions/
- http://imgur.com/r/math/vJX89



Anatoliĭ Fomenko

Born on March 13, 1945 in the Ukrainian city of Donetsk, Anatoliĭ Fomenko is the only child of a mining engineer and a teacher of Russian literature. Despite the brutality of the Stalinist years, his family made great efforts to ensure him an excellent education, and, in 1961, he entered the Mechanics–Mathematics Department at Moscow State University, where he has remained throughout his career.

To date, Fomenko has written more than 140 scientific publications, as well as 16 books and monographs. Much of his energies have also gone toward mathematics education: he is author of a wide variety of textbooks, especially in the area of modern geometry and topology.

Fomenko's fascination with art and drawing emerged when he was quite young, and he began creating paintings and sculpture as early as age 13. Since then, he has devoted a great deal of time to studying the work of many artistic masters. His work has been shown in exhibits around the world.

Topology and pastries

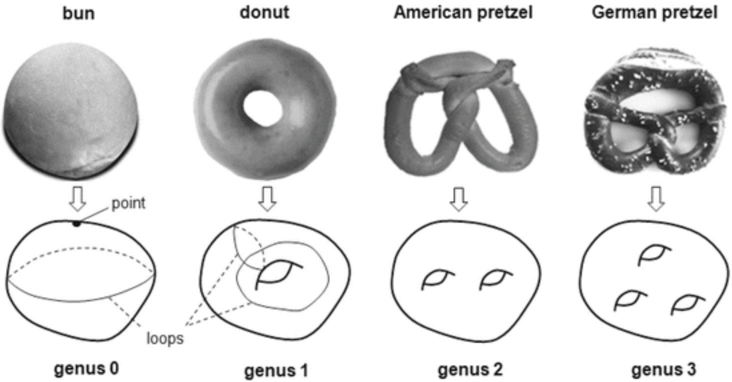
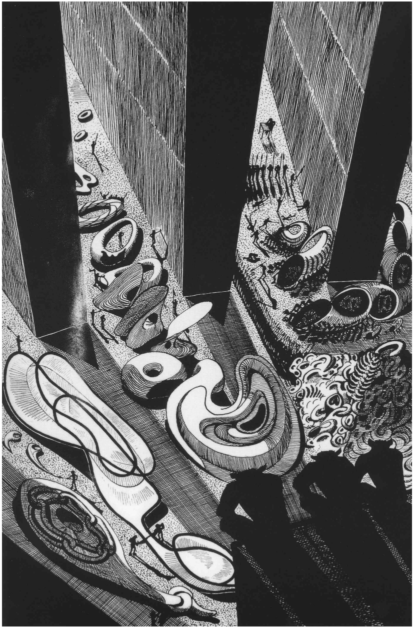


Image credit: Hung Nguyen-Schäfer, Jan-Philip Schmidt.



Topological zoo (1967)

Outline

Simplicial complexes

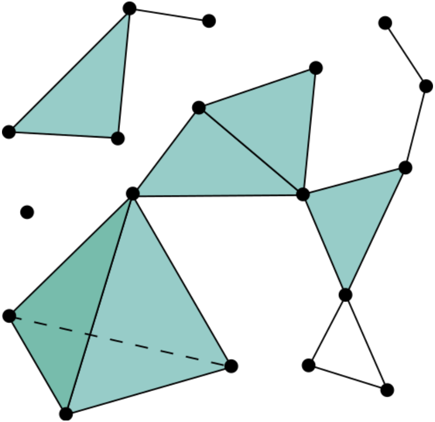
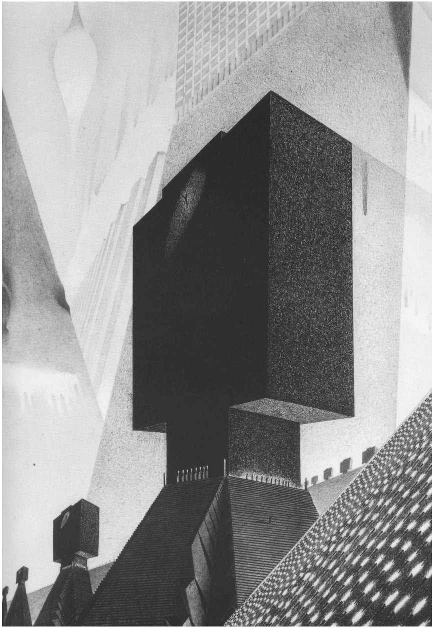
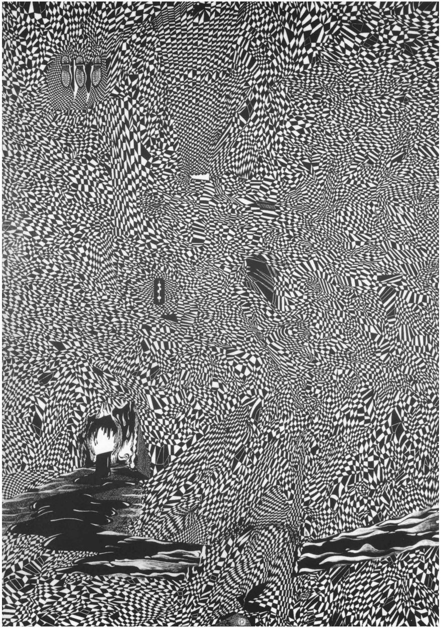


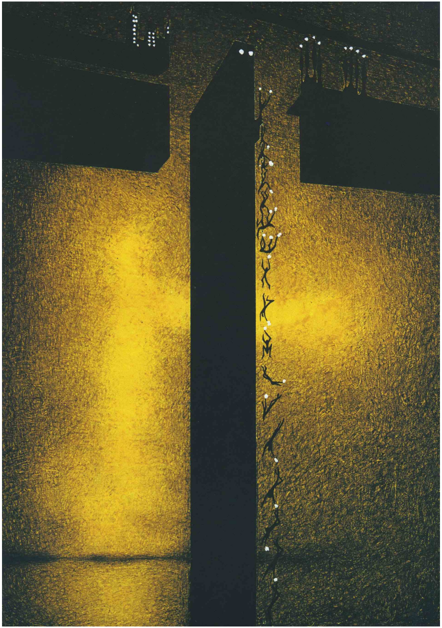
Figure: A simplicial complex of dimension 3.
Image credit: Wikipedia.



Simplicial complexes (1973)



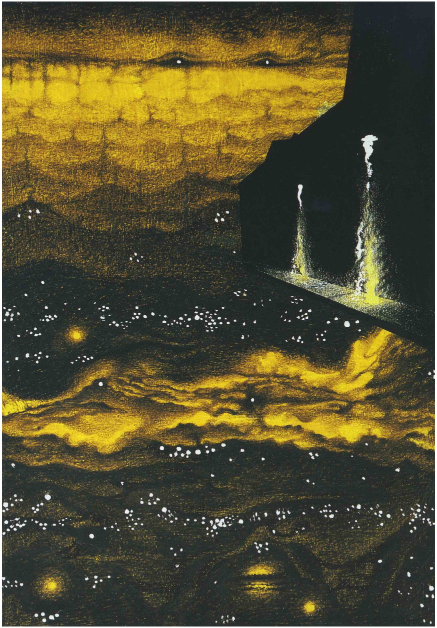
2-dimensional polyhedra and incidence matrices (1975)



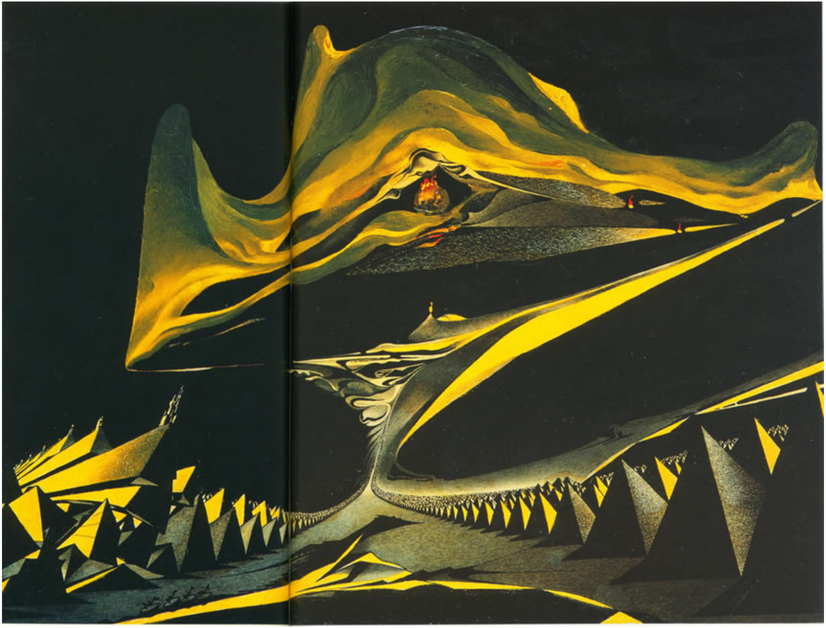
Construction of complicated polyhedra from simple ones, I (1972)

Construction of complicated polyhedra from simple ones, II (1972)

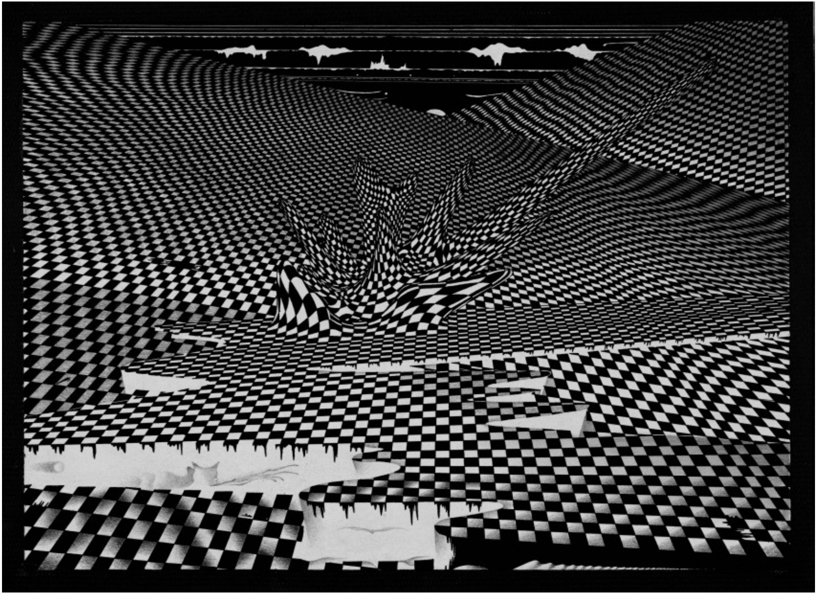
Construction of complicated polyhedra from simple ones, III (1972)



Construction of complicated polyhedra from simple ones, IV (1972)

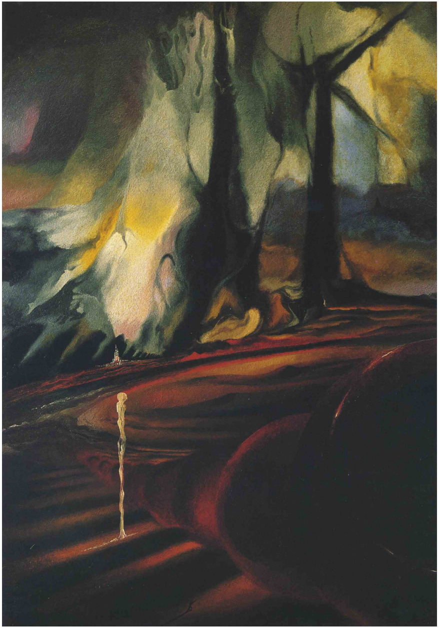


Combinatorial contraction (1973)

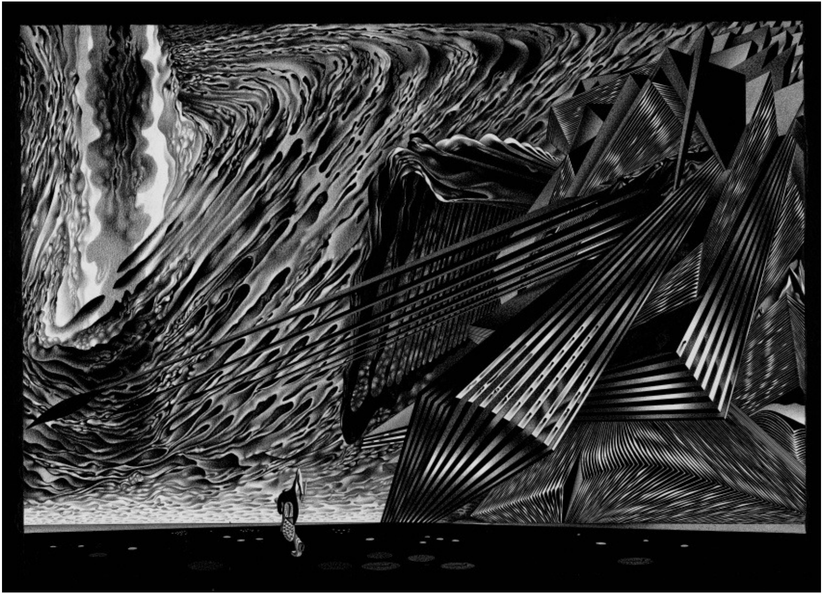


Algebraic surfaces of higher order and the simplicial approximation theorem (1974)

Outline

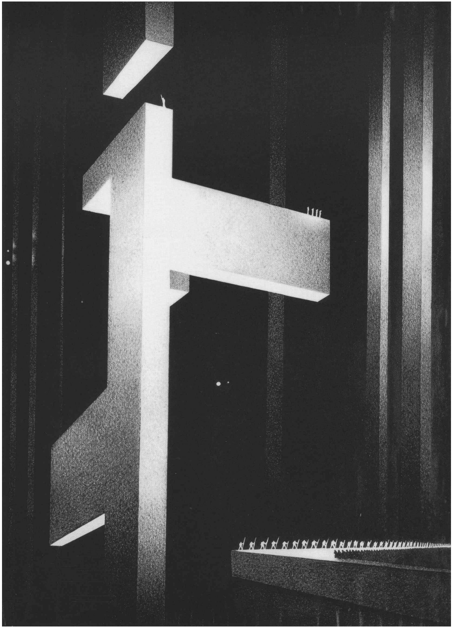


Cellular spaces (1970)

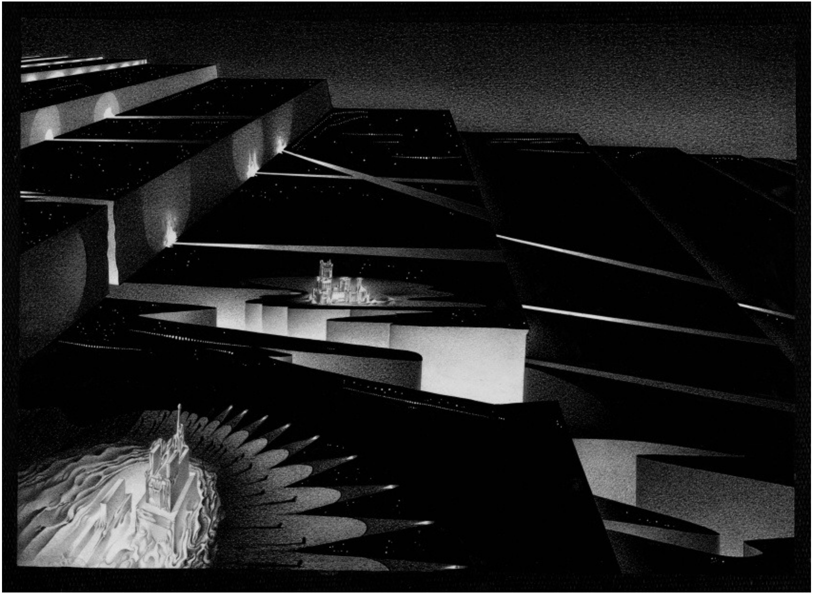


Simplicial spaces, cellular spaces, crystal and liquid (1976)

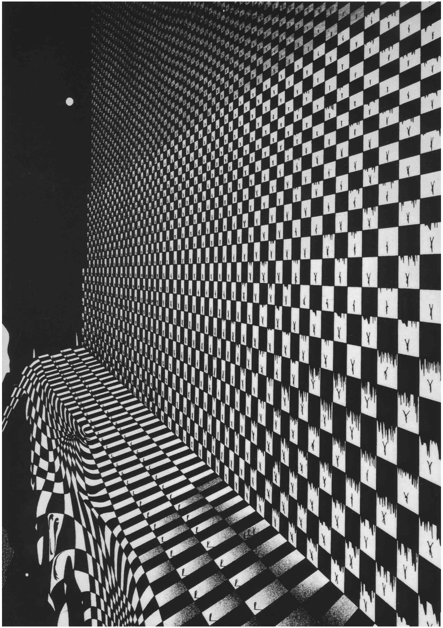
Outline



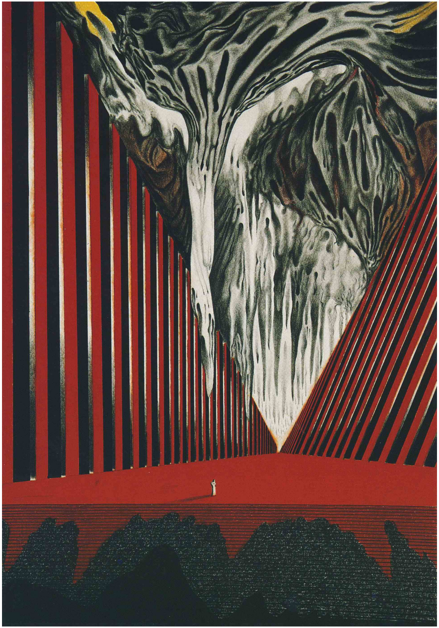
Polyhedra and simplicial chains (1973)



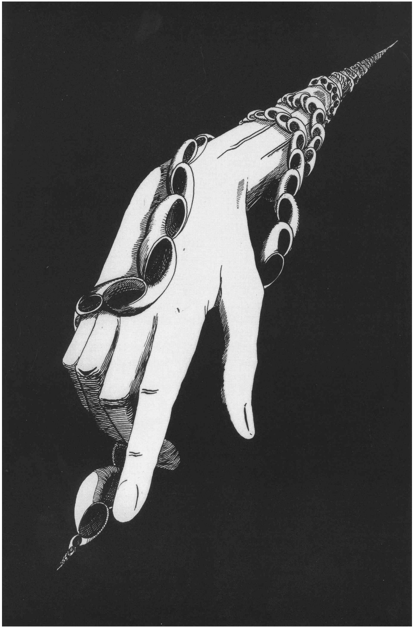
The boundary of polyhedra can be diminished when they are glued together (1973)



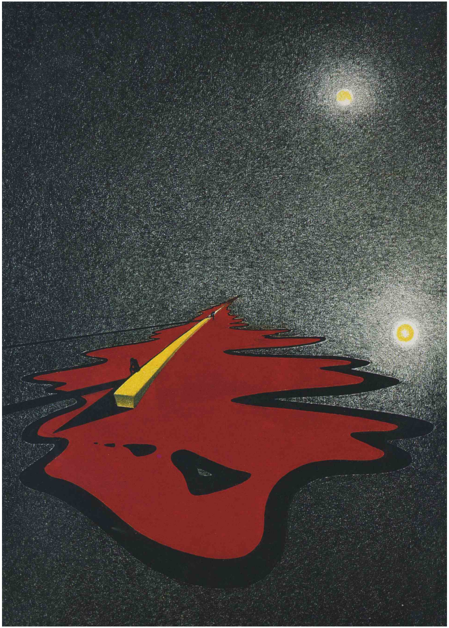
Simplicial, cubic, cellular chains (1974)



The theorem on the coincidence of simplicial and cellular homology (1973)



A space with nontrivial local homology (1967)



A system of shrinking neighborhoods (1973)

Outline

Homotopy groups

Homotopy groups of a space X:

$$\pi_k(X) = [S^k, X].$$

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Homotopy groups of spheres:

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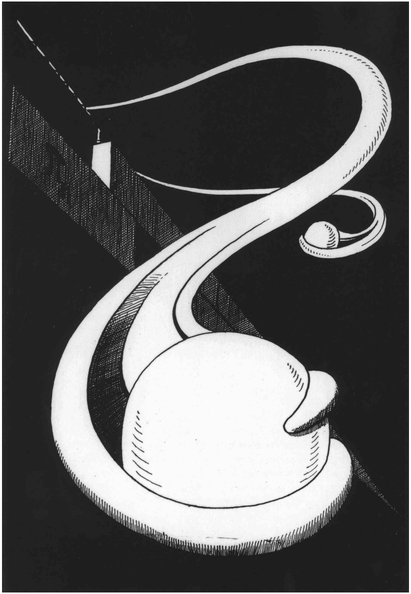
$$\pi_k(S^n) = [S^k, S^n].$$

Example.

$$\pi_k(S^1) = \begin{cases} \mathbb{Z} & \text{if } k = 1 \\ 0 & \text{if } k > 1. \end{cases}$$

Homotopy groups of spheres (1971)



The action of the fundamental group on the higher homotopy groups (1967)

The action of the fundamental group on the higher homotopy groups (1967)

The method of killing spaces in homotopic topology (1968)

Outline

Spectral sequences

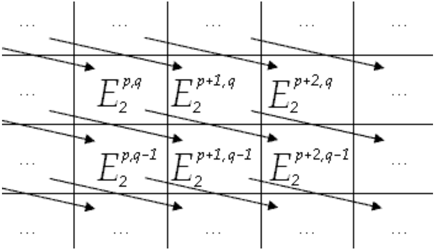
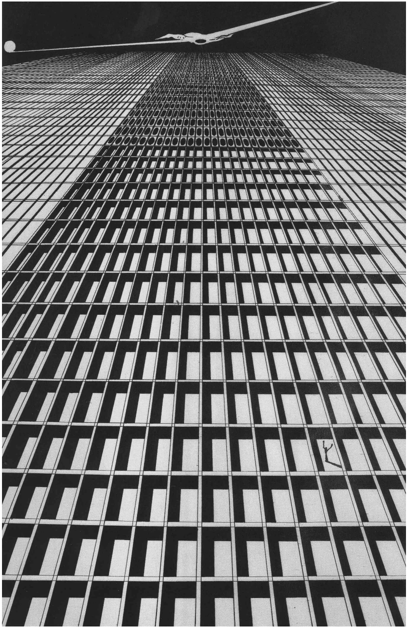
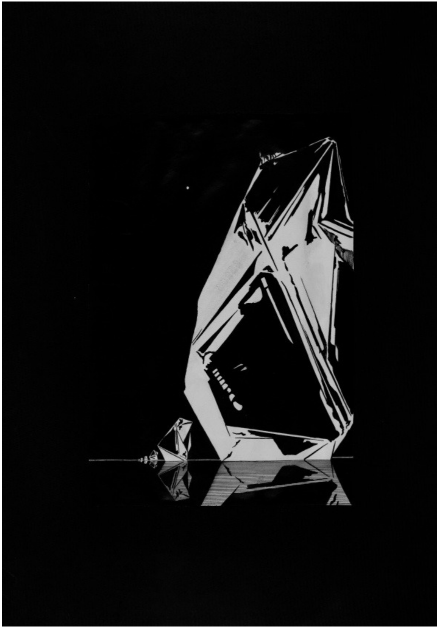


Figure: A spectral sequence.
Image credit: Wikipedia.



A spectral sequence (1967)

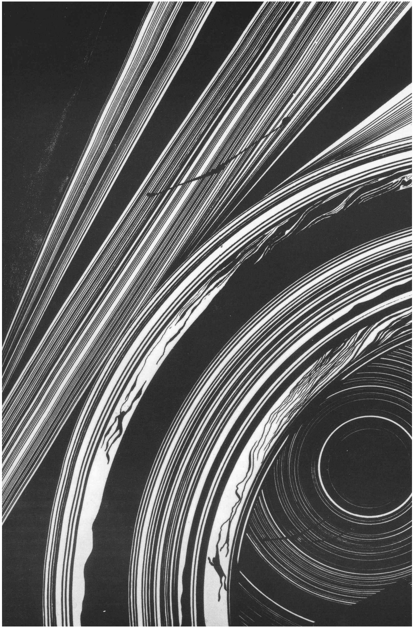


A spectral sequence (1967)

An Elizabethan drama

"The behavior of this spectral sequence... is a bit like an Elizabethan drama, full of action, in which the business of each character is to kill at least one other character, so that at the end of the play one has the stage strewn with corpses and only one actor left alive (namely the one who has to speak the last few lines)."

— J. Frank Adams



Spectral sequences and orbits of the action of groups (1967)

Outline

Morse theory

Morse function $f\colon X \to \mathbb{R}$.

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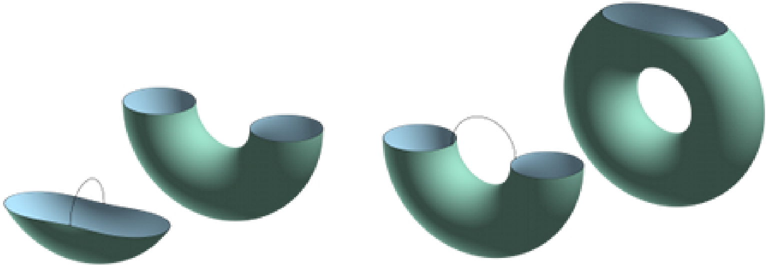
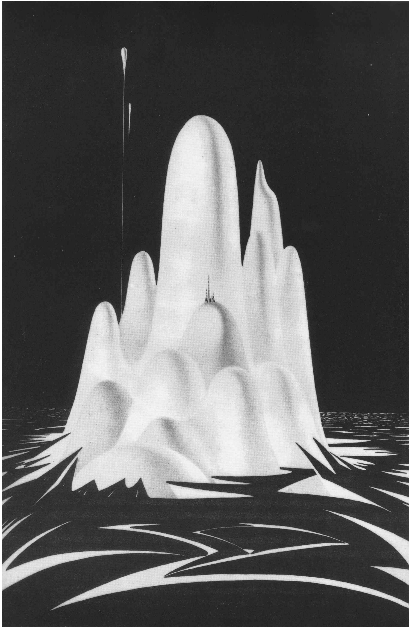


Figure: A Morse function on a torus.
Image credit: Wikipedia.



Morse functions and the theorem about the Euler characteristic (1975)

Topological restructuring of level surfaces of smooth functions on manifolds (1972)

Between two maxima there is always a saddle point (1968)

Outline

Knot theory

A knot $f\colon S^1 \hookrightarrow \mathbb{R}^3$.

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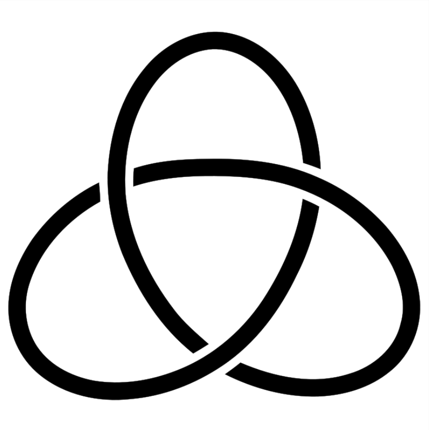
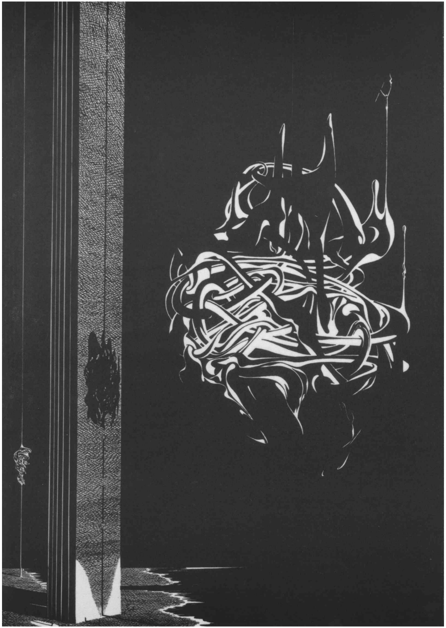


Figure: A trefoil knot.
Image credit: Wikipedia.



A nontrivial knot in 3-dimensional space (1974)

The Alexander horned sphere

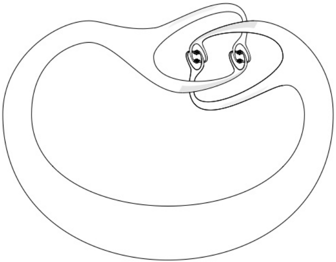
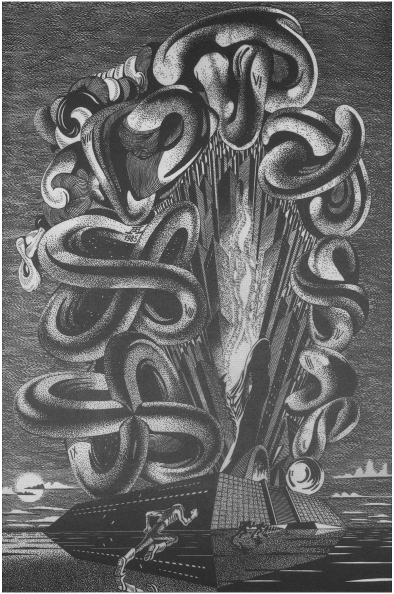


Figure: The Alexander horned sphere.
Image credit: Danny Calegari.

The Alexander horned sphere (1967)



A 2-dimensional sphere in 3-dimensional space can be turned inside out (1985)

A retraction of a space onto a subspace of it (1974)

Outline

Mapping cylinder

Continuous map $f\colon X \to Y$.

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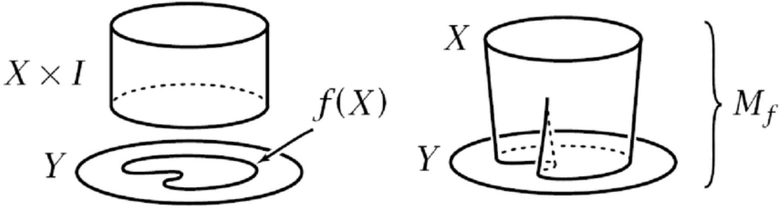
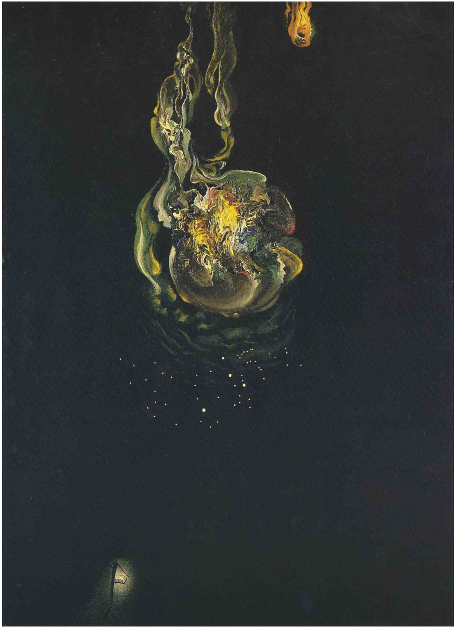


Figure: The mapping cylinder of f.
Image credit: Allen Hatcher.

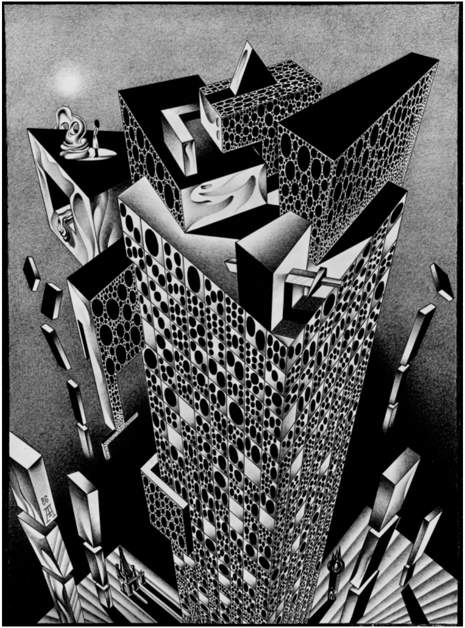
The cylinder of a continuous mapping (1967)

A fiber space (1970)

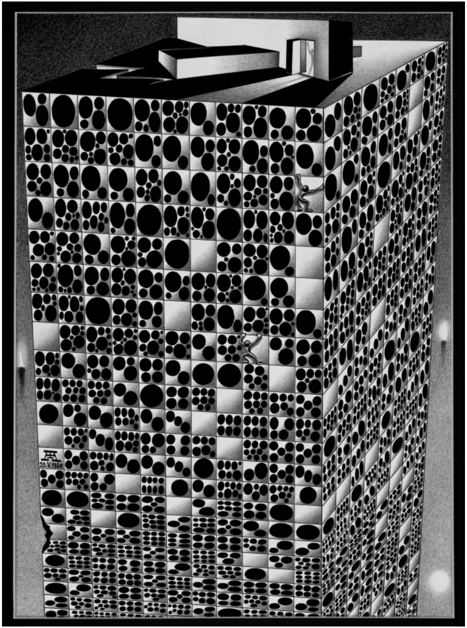


Branched coverings over a sphere (1976)

Outline



The remarkable numbers π and e, I (1986).



The remarkable numbers π *and e, II* (1986).

Thank you!