# DIRECTED READING PROJECTS

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Below are some topics that could serve as basis for a directed reading project, over one or two semesters. I find the mentioned results interesting, important, and beautiful. In each case, the goals of the project are the following.

- Understand the notions in and around the statement.
- Go through the proof and learn some of the techniques involved.
- Look at examples and applications of the statement.

## 1. Serre's finiteness theorem

**Theorem 1.1** (Serre)**.** *The homotopy group $\pi_{n+k}(S^n)$ is finite for $k > 0$ except for an even-dimensional sphere $n = 2m$ and $k = 2m - 1$, in which case we have $\pi_{4m-1}(S^{2m}) = \mathbb{Z} \oplus F_m$ for some finite group $F_m$.*

**Corollary 1.2.** *For every $k > 0$, the stable homotopy group $\pi_k^S$ is finite.*

References: [Ser53a], [Hat04, Theorem 1.21], [tD08, §20.8], [Rav04, §1.1].

## 2. The first few stable homotopy groups of spheres

**Theorem 2.1.** *The first few stable homotopy groups of spheres $\pi_k^S$ are as follows.*

| $k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $\pi_k^S$ | $\mathbb{Z}$ | $\mathbb{Z}/2$ | $\mathbb{Z}/2$ | $\mathbb{Z}/24$ | 0 | 0 | $\mathbb{Z}/2$ | $\mathbb{Z}/240$ |

We will follow Serre's method of iteratively getting rid of the bottom homotopy group and using the Serre spectral sequence.

References: [CS52], [Ser52], [Hat04, §1.Computing homotopy groups of spheres], [Rav04, §1.2].

*Remark* 2.2. Secretly, this project is an invitation to the Adams spectral sequence. Serre's method becomes unwieldy as the stem increases.

## 3. The Steenrod algebra and its dual

**Theorem 3.1** (Serre)**.** *The mod 2 Steenrod algebra is given as a graded $\mathbb{F}_2$-algebra by*

$$\mathcal{A}^* = T_{\mathbb{F}_2}(\mathrm{Sq}^1, \mathrm{Sq}^2, \ldots)/\textit{Adem relations}.$$

*Here,* $\mathrm{Sq}^i$ *denotes the* $i^{th}$ *Steenrod square, with degree* $|\mathrm{Sq}^i| = i$*.*

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**Theorem 3.2** (Cartan). *For an odd prime $p$, the mod $p$ Steenrod algebra is given as a graded $\mathbb{F}_p$-algebra by*

$$\mathcal{A}^* = T_{\mathbb{F}_p}(\beta, P^1, P^2, \ldots)/\textit{Adem relations}.$$

*Here, $P^i$ denotes the $i^{th}$ Steenrod reduced $p^{th}$ power, with degree $|P^i| = 2i(p-1)$, and $\beta$ denotes the mod $p$ Bockstein operation, with degree $|\beta| = 1$.*

**Theorem 3.3** (Milnor). *The mod 2 dual Steenrod algebra is a polynomial $\mathbb{F}_2$-algebra*

$$\mathcal{A}_* = \mathbb{F}_2[\xi_1, \xi_2, \ldots]$$

*on generators $\xi_i$ of degree $|\xi_i| = 2^i - 1$.*

*For $p$ an odd prime, the mod $p$ dual Steenrod algebra is free as a graded-commutative $\mathbb{F}_p$-algebra, namely*

$$\mathcal{A}_* = \mathbb{F}_p[\xi_1, \xi_2, \ldots] \otimes_{\mathbb{F}_p} \Lambda_{\mathbb{F}_p}[\tau_0, \tau_1, \ldots]$$

*on generators $\xi_i$ of degree $|\xi_i| = 2p^i - 2$ and $\tau_i$ of degree $|\tau_i| = 2p^i - 1$.*

References: [Ser53b], [Car55], [Mil58], [Hat02, §4.L], [MT68, §6], [Ada74, III.12], [May99, §22.5], [Mar83, §15].

## 4. The unoriented cobordism ring

**Theorem 4.1** (Thom). *The unoriented bordism ring is a polynomial $\mathbb{F}_2$-algebra*

$$\Omega_*^O \cong \pi_* MO = \mathbb{F}_2[u_2, u_4, u_5, u_6, u_8, u_9, \ldots]$$

*on generators $u_i$ of degree $|u_i| = i$ for all positive $i$ not of the form $2^k - 1$.*

References: [Tho54], [May99, §25], [tD08, §21].

## 5. The complex cobordism ring

**Theorem 5.1** (Milnor,Novikov). *The complex bordism ring is a polynomial ring*

$$\Omega_*^U \cong \pi_* MU = \mathbb{Z}[x_1, x_2, \ldots]$$

*on generators $x_i$ of degree $|x_i| = 2i$.*

Let $L$ denote the Lazard ring, which classifies formal group laws. That is, the data of a formal group law on a commutative ring $R$ is the same data as a ring homomorphism $L \to R$.

**Theorem 5.2** (Lazard). *The ring $L$ is a polynomial ring*

$$L \cong \mathbb{Z}[a_1, a_2, \ldots].$$

For an appropriate choice of grading, the generators have degree $|a_i| = i$. To match the grading appearing in topology, we should double this grading to $|a_i| = 2i$.

**Theorem 5.3** (Quillen). *The complex orientation of $MU$ induces the universal formal group law on $\pi_* MU$. In other words, the corresponding ring homomorphism $L \xrightarrow{\cong} \pi_* MU$ is an isomorphism.*

References: [Laz55], [Mil60], [Qui69], [Rav04, §1.2–1.3], [Ada74, §II.0-II.10], [KT06, §6].

## 6. The Bott periodicity theorem

**Theorem 6.1** (Bott). *The stable unitary group $U = \bigcup_n U(n)$ satisfies $\Omega^2 U \simeq U$. Its homotopy groups are given by $\pi_0 U = 0$, $\pi_1 U = \mathbb{Z}$, and the 2-periodicity.*

**Theorem 6.2** (Bott). *The stable orthogonal group $O = \bigcup_n O(n)$ satisfies $\Omega^8 O \simeq O$. Its homotopy groups are given by*

| $k$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $\pi_k O$ | $\mathbb{Z}/2$ | $\mathbb{Z}/2$ | 0 | $\mathbb{Z}$ | 0 | 0 | 0 | $\mathbb{Z}$ |

*and the 8-periodicity.*

References: [Bot59], [May99, §24.2], [Hat17, §2.2], [KT06, §4.2], [Ati67, §2.2], [Mil63, §23].

## 7. Simplicial sets as a model for spaces

**Theorem 7.1** (Quillen). *The geometric realization functor $|\cdot|$ and the singular set functor* Sing *form a Quillen equivalence*

$$|\cdot|\colon s\mathbf{Set} \rightleftarrows \mathbf{Top}\colon \mathrm{Sing}$$

*between simplicial sets (with the Kan–Quillen model structure) and topological spaces (with the Serre model structure).*

References: [Qui67, §II.3], [Qui68], [GJ09, §I.11], [Hov99, §3.6], [MP12, §17.5], [DS95, §8,11], [Hir03, §7.10, 8.5].

## 8. The Dold–Kan correspondence

**Theorem 8.1** (Dold,Kan). *The normalized chain complex functor*

$$N\colon s\mathbf{Ab} \to \mathbf{Ch}_{\geq 0}(\mathbb{Z})$$

*from simplicial abelian groups to non-negatively graded chain complexes of abelian groups is an equivalence of categories.*

Under this correspondence:

- The homotopy groups of a simplicial abelian group $A$ correspond to the homology groups of its normalized chain complex: $\pi_n A \cong H_n(NA)$.
- Simplicially homotopic maps $f, g\colon A \to B$ correspond to chain homotopic maps $Nf, Ng\colon NA \to NB$.

References: [Dol58, §1–2], [DP61, §3], [Wei94, §8.4], [GJ09, §III.2].

## 9. The recognition principle for loop spaces

**Theorem 9.1** (Stasheff, May, Boardman–Vogt). *A topological space $X$ is weakly equivalent to a loop space $\Omega Y$ if and only if it admits the structure of a group-like $A_\infty$ space.*

References: [Sta70, §11], [May72, §3, 13], [BV73, §I.1–I.4, VI.1–VI.3], [MSS02, §I.1.6, II.2.1–II.2.3].

## 10. Universal algebra from a categorical viewpoint

**Theorem 10.1** (Lawvere)**.** *The following properties of a category $\mathcal{C}$ are equivalent: $\mathcal{C}$ is equivalent to...*

(1) *A (one-sorted) finitary variety of algebras.*
(2) *The category of finite product preserving functors $\mathcal{T} \to$ **Set** for a (one-sorted) Lawvere theory $\mathcal{T}$.*
(3) *The category of algebras for a finitary monad on **Set**.*
(4) *A cocomplete category with a small projective generator and effective equivalence relations.*

References: [Law63], [AR94, §3.A], [ARV11, §1,6], [ML98, §V.6, VI.8], [Bor94, §3.1–3.9, 4.1–4.4].

## 11. Algebraic $K$-theory

Let $R$ be a ring. The $0^{\text{th}}$ algebraic $K$-group $K_0(R)$ is defined as the Grothendieck group of finitely generated projective $R$-modules.

**Definition 11.1** (Quillen)**.** For $n > 0$, the *algebraic $K$-groups* of $R$ are defined by

$$K_n(R) = \pi_n\left(BGL(R)^+\right).$$

*Remark* 11.2. This definition recovers the group $K_1(R)$ previously defined by Bass [Bas64] and the group $K_2(R)$ defined by Milnor [Mil71].

Covering the ingredients that go into this definition would provide enough material for a project. A more ambitious project could also tackle the following result.

**Theorem 11.3** (Quillen)**.** *Let $\mathbb{F}_q$ be a finite field of cardinality $q$. Then the algebraic $K$-groups of $\mathbb{F}_q$ are:*

$$\begin{cases} K_0(\mathbb{F}_q) = \mathbb{Z} & \\ K_{2i}(\mathbb{F}_q) = 0 & \text{for } i \geq 1 \\ K_{2i-1}(\mathbb{F}_q) = \mathbb{Z}/(q^i - 1) & \text{for } i \geq 1. \end{cases}$$

References: [Qui73], [Qui75], [Ros94, §5], [Wei13, §IV.1], [Wei99].

## 12. Stokes' theorem and De Rham cohomology

**Theorem 12.1** (Stokes' theorem)**.** *Let $M$ be a compact oriented $n$-dimensional smooth manifold with boundary $\partial M$. Let $\omega$ be an $(n-1)$-form on $M$. Then we have*

$$\int_M d\omega = \int_{\partial M} \omega,$$

*where $d\omega$ denotes the exterior derivative of $\omega$.*

**Theorem 12.2** (de Rham)**.** *Let $M$ be an $n$-dimensional smooth manifold. There is a natural isomorphism*

$$H^*_{dR}(M) \cong H^*(M; \mathbb{R})$$

*between de Rham cohomology of $M$ and singular cohomology of $M$ with real coefficients.*

*Remark* 12.3. Armed with those two theorems, it is fun to revisit vector calculus in dimensions 2 and 3. Gradient, curl, and divergence are the differentials in the de Rham complex.

References: [Bre97, §V.1–V.5, V.9], [War83, §4,5], [Rud76, §10] [BT82, §1–5, 8] [GP10, §4.1–4.7], [Tao18].

## 13. The Morse homology theorem

**Theorem 13.1** (Smale). *Let $M$ be a closed smooth manifold and $f\colon M \to \mathbb{R}$ a Morse function. Then $M$ admits a handle decomposition with one $k$-handle for each critical point of $f$ of index $k$.*

See [Sma61, §6], [Pal63, §12], [Kos93, §VII.2].

**Theorem 13.2.** *Let $M$ be a closed oriented smooth manifold, $f\colon M \to \mathbb{R}$ a Morse function, and $g$ a Riemannian metric on $M$ making the pair $(f, g)$ Morse–Smale. Then there is a canonical isomorphism*

$$H_*^{\mathrm{Morse}}(f, g) \cong H_*(M; \mathbb{Z})$$

*between the Morse homology of the pair $(f, g)$ and the singular homology of $M$.*

*Remark* 13.3. With more work, one can remove the assumption that $M$ be orientable.

References: [Mil63, §1], [Mat02], [Hir94, §6], [Sch93, §4], [Hut02, §1–4], [BH04], [Nic11, §1–2].

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