

# DIRECTED READING PROJECTS

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Below are some topics that could serve as basis for a directed reading project, over one or two semesters. I find the mentioned results interesting, important, and beautiful. In each case, the goals of the project are the following.

- Understand the notions in and around the statement.
- Go through the proof and learn some of the techniques involved.
- Look at examples and applications of the statement.

## 1. SERRE'S FINITENESS THEOREM

**Theorem 1.1** (Serre). *The homotopy group  $\pi_{n+k}(S^n)$  is finite for  $k > 0$  except for an even-dimensional sphere  $n = 2m$  and  $k = 2m - 1$ , in which case we have  $\pi_{4m-1}(S^{2m}) = \mathbb{Z} \oplus F_m$  for some finite group  $F_m$ .*

**Corollary 1.2.** *For every  $k > 0$ , the stable homotopy group  $\pi_k^S$  is finite.*

References: [Ser53a], [Hat04, Theorem 1.21], [tD08, §20.8], [Rav04, §1.1], [McC01, §5], [Wei94, §5.3].

## 2. THE FIRST FEW STABLE HOMOTOPY GROUPS OF SPHERES

**Theorem 2.1.** *The first few stable homotopy groups of spheres  $\pi_k^S$  are as follows.*

$k$	0	1	2	3	4	5	6	7
$\pi_k^S$	$\mathbb{Z}$	$\mathbb{Z}/2$	$\mathbb{Z}/2$	$\mathbb{Z}/24$	0	0	$\mathbb{Z}/2$	$\mathbb{Z}/240$

We will follow Serre's method of iteratively getting rid of the bottom homotopy group and using the Serre spectral sequence.

References: [CS52], [Ser52], [Hat04, §1.Computing homotopy groups of spheres], [Rav04, §1.2], [McC01, §6].

*Remark 2.2.* Secretly, this project is an invitation to the Adams spectral sequence. Serre's method becomes unwieldy as the stem increases.

## 3. THE STEENROD ALGEBRA AND ITS DUAL

**Theorem 3.1** (Serre). *The mod 2 Steenrod algebra is given as a graded  $\mathbb{F}_2$ -algebra by*

$$\mathcal{A}^* = T_{\mathbb{F}_2}(\mathrm{Sq}^1, \mathrm{Sq}^2, \dots) / \text{Adem relations.}$$

Here,  $\mathrm{Sq}^i$  denotes the  $i^{\text{th}}$  Steenrod square, with degree  $|\mathrm{Sq}^i| = i$ .

**Theorem 3.2** (Cartan). *For an odd prime  $p$ , the mod  $p$  Steenrod algebra is given as a graded  $\mathbb{F}_p$ -algebra by*

$$\mathcal{A}^* = T_{\mathbb{F}_p}(\beta, P^1, P^2, \dots) / \text{Adem relations.}$$

Here,  $P^i$  denotes the  $i^{\text{th}}$  Steenrod reduced  $p^{\text{th}}$  power, with degree  $|P^i| = 2i(p-1)$ , and  $\beta$  denotes the mod  $p$  Bockstein operation, with degree  $|\beta| = 1$ .

**Theorem 3.3** (Milnor). *The mod 2 dual Steenrod algebra is a polynomial  $\mathbb{F}_2$ -algebra*

$$\mathcal{A}_* = \mathbb{F}_2[\xi_1, \xi_2, \dots]$$

on generators  $\xi_i$  of degree  $|\xi_i| = 2^i - 1$ .

For  $p$  an odd prime, the mod  $p$  dual Steenrod algebra is free as a graded-commutative  $\mathbb{F}_p$ -algebra, namely

$$\mathcal{A}_* = \mathbb{F}_p[\xi_1, \xi_2, \dots] \otimes_{\mathbb{F}_p} \Lambda_{\mathbb{F}_p}[\tau_0, \tau_1, \dots]$$

on generators  $\xi_i$  of degree  $|\xi_i| = 2p^i - 2$  and  $\tau_i$  of degree  $|\tau_i| = 2p^i - 1$ .

References: [Ser53b], [Car55], [Mil58], [Hat02, §4.L], [MT68, §6], [Ada74, III.12], [May99, §22.5], [Mar83, §15].

#### 4. THE UNORIENTED COBORDISM RING

**Theorem 4.1** (Thom). *The unoriented bordism ring is a polynomial  $\mathbb{F}_2$ -algebra*

$$\Omega_*^O \cong \pi_* MO = \mathbb{F}_2[u_2, u_4, u_5, u_6, u_8, u_9, \dots]$$

on generators  $u_i$  of degree  $|u_i| = i$  for all positive  $i$  not of the form  $2^k - 1$ .

References: [Tho54], [May99, §25], [tD08, §21].

#### 5. THE COMPLEX COBORDISM RING

**Theorem 5.1** (Milnor, Novikov). *The complex bordism ring is a polynomial ring*

$$\Omega_*^U \cong \pi_* MU = \mathbb{Z}[x_1, x_2, \dots]$$

on generators  $x_i$  of degree  $|x_i| = 2i$ .

Let  $L$  denote the Lazard ring, which classifies formal group laws. That is, the data of a formal group law on a commutative ring  $R$  is the same data as a ring homomorphism  $L \rightarrow R$ .

**Theorem 5.2** (Lazard). *The ring  $L$  is a polynomial ring*

$$L \cong \mathbb{Z}[a_1, a_2, \dots].$$

For an appropriate choice of grading, the generators have degree  $|a_i| = i$ . To match the grading appearing in topology, we should double this grading to  $|a_i| = 2i$ .

**Theorem 5.3** (Quillen). *The complex orientation of  $MU$  induces the universal formal group law on  $\pi_* MU$ . In other words, the corresponding ring homomorphism  $L \xrightarrow{\cong} \pi_* MU$  is an isomorphism.*

References: [Laz55], [Mil60], [Qui69], [Rav04, §1.2–1.3], [Ada74, §II.0–II.10], [KT06, §6].

## 6. THE BOTT PERIODICITY THEOREM

**Theorem 6.1** (Bott). *The stable unitary group  $U = \bigcup_n U(n)$  satisfies  $\Omega^2 U \simeq U$ . Its homotopy groups are given by  $\pi_0 U = 0$ ,  $\pi_1 U = \mathbb{Z}$ , and the 2-periodicity.*

**Theorem 6.2** (Bott). *The stable orthogonal group  $O = \bigcup_n O(n)$  satisfies  $\Omega^8 O \simeq O$ . Its homotopy groups are given by*

$k$	0	1	2	3	4	5	6	7
$\pi_k O$	$\mathbb{Z}/2$	$\mathbb{Z}/2$	0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$

and the 8-periodicity.

References: [Bot59], [May99, §24.2], [Hat17, §2.2], [KT06, §4.2], [Ati67, §2.2], [Mil63, §23].

## 7. SIMPLICIAL SETS AS A MODEL FOR SPACES

**Theorem 7.1** (Quillen). *The geometric realization functor  $|\cdot|$  and the singular set functor  $\text{Sing}$  form a Quillen equivalence*

$$|\cdot|: \mathbf{sSet} \rightleftarrows \mathbf{Top}: \text{Sing}$$

between simplicial sets (with the Kan–Quillen model structure) and topological spaces (with the Serre model structure).

References: [Qui67, §II.3], [Qui68], [GJ09, §I.11], [Hov99, §3.6], [MP12, §17.5], [DS95, §8,11], [Hir03, §7.10, 8.5].

## 8. THE DOLD–KAN CORRESPONDENCE

**Theorem 8.1** (Dold, Kan). *The normalized chain complex functor*

$$N: \mathbf{sAb} \rightarrow \mathbf{Ch}_{\geq 0}(\mathbb{Z})$$

from simplicial abelian groups to non-negatively graded chain complexes of abelian groups is an equivalence of categories.

Under this correspondence:

- The homotopy groups of a simplicial abelian group  $A$  correspond to the homology groups of its normalized chain complex:  $\pi_n A \cong H_n(NA)$ .
- Simplicially homotopic maps  $f, g: A \rightarrow B$  correspond to chain homotopic maps  $Nf, Ng: NA \rightarrow NB$ .

References: [Dol58, §1–2], [DP61, §3], [Wei94, §8.4], [GJ09, §III.2].

## 9. THE RECOGNITION PRINCIPLE FOR LOOP SPACES

**Theorem 9.1** (Stasheff, May, Boardman–Vogt). *A topological space  $X$  is weakly equivalent to a loop space  $\Omega Y$  if and only if it admits the structure of a group-like  $A_\infty$  space.*

References: [Sta70, §11], [May72, §3, 13], [BV73, §I.1–I.4, VI.1–VI.3], [MSS02, §I.1.6, II.2.1–II.2.3].

## 10. UNIVERSAL ALGEBRA FROM A CATEGORICAL VIEWPOINT

**Theorem 10.1** (Lawvere). *The following properties of a category  $\mathcal{C}$  are equivalent:  $\mathcal{C}$  is equivalent to...*

- (1) *A (one-sorted) finitary variety of algebras.*
- (2) *The category of finite product preserving functors  $\mathcal{T} \rightarrow \mathbf{Set}$  for a (one-sorted) Lawvere theory  $\mathcal{T}$ .*
- (3) *The category of algebras for a finitary monad on  $\mathbf{Set}$ .*
- (4) *A cocomplete category with a small projective generator and effective equivalence relations.*

References: [Law63], [AR94, §3.A], [ARV11, §1,6], [ML98, §V.6, VI.8], [Bor94, §3.1–3.9, 4.1–4.4].

11. ALGEBRAIC  $K$ -THEORY

Let  $R$  be a ring. The 0<sup>th</sup> algebraic  $K$ -group  $K_0(R)$  is defined as the Grothendieck group of finitely generated projective  $R$ -modules.

**Definition 11.1** (Quillen). For  $n > 0$ , the *algebraic  $K$ -groups* of  $R$  are defined by

$$K_n(R) = \pi_n(BGL(R)^+).$$

*Remark 11.2.* This definition recovers the group  $K_1(R)$  previously defined by Bass [Bas64] and the group  $K_2(R)$  defined by Milnor [Mil71].

Covering the ingredients that go into this definition would provide enough material for a project. A more ambitious project could also tackle the following result.

**Theorem 11.3** (Quillen). *Let  $\mathbb{F}_q$  be a finite field of cardinality  $q$ . Then the algebraic  $K$ -groups of  $\mathbb{F}_q$  are:*

$$\begin{cases} K_0(\mathbb{F}_q) = \mathbb{Z} \\ K_{2i}(\mathbb{F}_q) = 0 \\ K_{2i-1}(\mathbb{F}_q) = \mathbb{Z}/(q^i - 1) \end{cases} \quad \text{for } i \geq 1.$$

References: [Qui73], [Qui75], [Ros94, §5], [Wei13, §IV.1], [Wei99].

## 12. STOKES' THEOREM AND DE RHAM COHOMOLOGY

**Theorem 12.1** (Stokes' theorem). *Let  $M$  be a compact oriented  $n$ -dimensional smooth manifold with boundary  $\partial M$ . Let  $\omega$  be an  $(n-1)$ -form on  $M$ . Then we have*

$$\int_M d\omega = \int_{\partial M} \omega,$$

where  $d\omega$  denotes the exterior derivative of  $\omega$ .

**Theorem 12.2** (de Rham). *Let  $M$  be an  $n$ -dimensional smooth manifold. There is a natural isomorphism*

$$H_{dR}^*(M) \cong H^*(M; \mathbb{R})$$

between de Rham cohomology of  $M$  and singular cohomology of  $M$  with real coefficients.

*Remark 12.3.* Armed with those two theorems, it is fun to revisit vector calculus in dimensions 2 and 3. Gradient, curl, and divergence are the differentials in the de Rham complex.

References: [Bre97, §V.1–V.5, V.9], [War83, §4,5], [Rud76, §10] [BT82, §1–5, 8] [GP10, §4.1–4.7], [Tao18].

### 13. THE MORSE HOMOLOGY THEOREM

**Theorem 13.1** (Smale). *Let  $M$  be a closed smooth manifold and  $f: M \rightarrow \mathbb{R}$  a Morse function. Then  $M$  admits a handle decomposition with one  $k$ -handle for each critical point of  $f$  of index  $k$ .*

See [Sma61, §6], [Pal63, §12], [Kos93, §VII.2].

**Theorem 13.2.** *Let  $M$  be a closed oriented smooth manifold,  $f: M \rightarrow \mathbb{R}$  a Morse function, and  $g$  a Riemannian metric on  $M$  making the pair  $(f, g)$  Morse–Smale. Then there is a canonical isomorphism*

$$H_*^{\text{Morse}}(f, g) \cong H_*(M; \mathbb{Z})$$

*between the Morse homology of the pair  $(f, g)$  and the singular homology of  $M$ .*

*Remark 13.3.* With more work, one can remove the assumption that  $M$  be orientable.

References: [Mil63, §1], [Mat02], [Hir94, §6], [Sch93, §4], [Hut02, §1–4], [BH04], [Nic11, §1–2].

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