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**Energy eigenvalues:**

Particle in a box: 
$$E_n = \frac{n^2 h^2}{8mL^2}$$

Harmonic oscillator: 
$$E_n = h\nu_0\left(n + \frac{1}{2}\right), \quad \nu_0 = \frac{1}{2\pi} \sqrt{\frac{k}{\mu}}$$

Rigid rotor (Particle on a sphere): 
$$E_{lm} = \frac{\hbar^2}{2I} l(l+1), \quad I = \mu r^2$$

One-electron atom: 
$$E_{nlm} = -hc\mathfrak{R} \frac{Z^2}{n^2}, \quad \mathfrak{R} = \frac{\mu e^4}{8h^3 c \epsilon_0^2}$$

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**Energy eigenfunctions:**

Particle in a box: 
$$\Psi_n(x) = N_n \sin\left(\frac{n\pi x}{L}\right), \quad N_n = \left[\frac{2}{L}\right]^{1/2}$$

Harmonic oscillator: 
$$\Psi_n(x) = N_n H_n(y) e^{-y^2/2}, \quad y = \frac{1}{\alpha} x, \quad \alpha = \left(\frac{\hbar^2}{k\mu}\right)^{1/4}, \quad N_n = \left[\frac{1}{\alpha\pi^{1/2} 2^n n!}\right]^{1/2}$$

examples: 
$$\Psi_0(x) = \left[\frac{1}{\alpha\pi^{1/2}}\right]^{1/2} e^{-x^2/2\alpha^2}, \quad \Psi_1(x) = \left[\frac{1}{\alpha\pi^{1/2} 2}\right]^{1/2} \frac{2}{\alpha} x e^{-x^2/2\alpha^2}$$

Hermite polynomials: 
$$H_0(y) = 1, \quad H_1(y) = 2y, \quad H_2(y) = 4y^2 - 2, \quad H_3(y) = 8y^3 - 12y$$

Rigid rotor: 
$$Y_{lm}(\theta, \phi) = N_{lm} P_l^{|m|}(u) e^{im\phi}, \quad u = \cos\theta$$

examples: 
$$Y_{1,\pm 1}(\theta, \phi) = \mp \sqrt{\frac{3}{8\pi}} (\sin\theta) (e^{\pm i\phi}), \quad Y_{2,0}(\theta, \phi) = \sqrt{\frac{5}{16\pi}} (3\cos^2\theta - 1)$$

Radial wave function: 
$$R_{nl}(r) = N_{nl} \rho^l L_{n+l}^{2l+1}(\rho) e^{-\rho/2}, \quad \rho = \frac{2Z}{na_0} r$$

examples: 
$$R_{1,0}(r) = 2(Z/a_0)^{3/2} e^{-\rho/2},$$
$$R_{2,0}(r) = 8^{-1/2} (Z/a_0)^{3/2} (2 - \rho) e^{-\rho/2},$$
$$R_{2,1}(r) = 24^{-1/2} (Z/a_0)^{3/2} \rho e^{-\rho/2},$$
$$R_{3,0}(r) = 243^{-1/2} (Z/a_0)^{3/2} (6 - 6\rho + \rho^2) e^{-\rho/2}$$

One-electron atom: 
$$\Psi_{nlm}(r, \theta, \phi) = R_{nl}(r) Y_{lm}(\theta, \phi)$$

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**Fundamental constants:**

$h = 6.62608 \times 10^{-34} \text{ J s}$	$\text{amu} = 1.66054 \times 10^{-27} \text{ kg}$	$c = 2.99792 \times 10^{10} \text{ cm s}^{-1}$
$\hbar = 1.05457 \times 10^{-34} \text{ J s}$	$m_e = 9.10939 \times 10^{-31} \text{ kg}$	$\mathfrak{R}_H = 1.09677 \times 10^5 \text{ cm}^{-1}$
$a_0 = 5.29177 \times 10^{-11} \text{ m}$	$m_h = 1.67493 \times 10^{-27} \text{ kg}$	$\mathfrak{R}_\infty = 1.09737 \times 10^5 \text{ cm}^{-1}$
$k = 1.38066 \times 10^{-23} \text{ J K}^{-1}$	$m_p = 1.67262 \times 10^{-27} \text{ kg}$	$N_{\text{AVO}} = 6.02214 \times 10^{23}$

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**Other quantities:**

Operators:  $\hat{J}_z = -i\hbar \frac{\partial}{\partial \phi}$

$$\hat{J}^2 = -\hbar^2 \left[ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right]$$

$$\bar{\nabla}^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2}$$

Trigonometric identities:

$$\begin{aligned} \sin \alpha \sin \beta &= \frac{1}{2} \cos(\alpha - \beta) - \frac{1}{2} \cos(\alpha + \beta) \\ \cos \alpha \cos \beta &= \frac{1}{2} \cos(\alpha - \beta) + \frac{1}{2} \cos(\alpha + \beta) \\ \sin \alpha \cos \beta &= \frac{1}{2} \sin(\alpha - \beta) + \frac{1}{2} \sin(\alpha + \beta) \\ e^{\pm i\theta} &= \cos \theta \pm i \sin \theta \end{aligned}$$

Integrals (n denotes positive integer):

$$\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{4a}}$$

$$\int_0^{\infty} x^{2n} e^{-ax^2} dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1} a^n} \sqrt{\frac{\pi}{a}}$$

$$\int_0^{\infty} x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

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## Periodic Table of the Elements 2016

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 <b>H</b> 1.008																	2 <b>He</b> 4.0026
3 <b>Li</b> 6.94	4 <b>Be</b> 9.0122											5 <b>B</b> 10.81	6 <b>C</b> 12.011	7 <b>N</b> 14.007	8 <b>O</b> 15.999	9 <b>F</b> 18.998	10 <b>Ne</b> 20.18
11 <b>Na</b> 22.99	12 <b>Mg</b> 24.305											13 <b>Al</b> 26.982	14 <b>Si</b> 28.085	15 <b>P</b> 30.974	16 <b>S</b> 32.06	17 <b>Cl</b> 35.45	18 <b>Ar</b> 39.948
19 <b>K</b> 39.098	20 <b>Ca</b> 40.078	21 <b>Sc</b> 44.956	22 <b>Ti</b> 47.867	23 <b>V</b> 50.942	24 <b>Cr</b> 51.996	25 <b>Mn</b> 54.938	26 <b>Fe</b> 55.845	27 <b>Co</b> 58.933	28 <b>Ni</b> 58.693	29 <b>Cu</b> 63.546	30 <b>Zn</b> 65.38	31 <b>Ga</b> 69.723	32 <b>Ge</b> 72.63	33 <b>As</b> 74.922	34 <b>Se</b> 78.96	35 <b>Br</b> 79.904	36 <b>Kr</b> 83.798
37 <b>Rb</b> 85.468	38 <b>Sr</b> 87.62	39 <b>Y</b> 88.906	40 <b>Zr</b> 91.224	41 <b>Nb</b> 92.906	42 <b>Mo</b> 95.96	43 <b>Tc</b> [97.91]	44 <b>Ru</b> 101.07	45 <b>Rh</b> 102.91	46 <b>Pd</b> 106.42	47 <b>Ag</b> 107.87	48 <b>Cd</b> 112.41	49 <b>In</b> 114.82	50 <b>Sn</b> 118.71	51 <b>Sb</b> 121.76	52 <b>Te</b> 127.6	53 <b>I</b> 126.9	54 <b>Xe</b> 131.29
55 <b>Cs</b> 132.91	56 <b>Ba</b> 137.33	* [226.03]	72 <b>Hf</b> 178.49	73 <b>Ta</b> 180.95	74 <b>W</b> 183.84	75 <b>Re</b> 186.21	76 <b>Os</b> 190.23	77 <b>Ir</b> 192.22	78 <b>Pt</b> 195.08	79 <b>Au</b> 196.97	80 <b>Hg</b> 200.59	81 <b>Tl</b> 204.38	82 <b>Pb</b> 207.2	83 <b>Bi</b> 208.98	84 <b>Po</b> [208.98]	85 <b>At</b> [209.99]	86 <b>Rn</b> [222.02]
87 <b>Fr</b> [223.02]	88 <b>Ra</b> [226.03]	** [226.03]	104 <b>Rf</b> [265.12]	105 <b>Db</b> [268.13]	106 <b>Sg</b> [271.13]	107 <b>Bh</b> [270]	108 <b>Hs</b> [277.15]	109 <b>Mt</b> [276.15]	110 <b>Ds</b> [281.16]	111 <b>Rg</b> [280.16]	112 <b>Cn</b> [285.17]	113 <b>Nh</b> [284.18]	114 <b>Fl</b> [289.19]	115 <b>Mc</b> [288.19]	116 <b>Lv</b> [293]	117 <b>Ts</b> [294]	118 <b>Og</b> [294]
<b>Lanthanoids</b>		57 <b>La</b> 138.91	58 <b>Ce</b> 140.12	59 <b>Pr</b> 140.91	60 <b>Nd</b> 144.24	61 <b>Pm</b> [144.91]	62 <b>Sm</b> 150.36	63 <b>Eu</b> 151.96	64 <b>Gd</b> 157.25	65 <b>Tb</b> 158.93	66 <b>Dy</b> 162.5	67 <b>Ho</b> 164.93	68 <b>Er</b> 167.26	69 <b>Tm</b> 168.93	70 <b>Yb</b> 173.05	71 <b>Lu</b> 174.97	
<b>Actinoids</b>		89 <b>Ac</b> [227.03]	90 <b>Th</b> 232.04	91 <b>Pa</b> 231.04	92 <b>U</b> 238.03	93 <b>Np</b> [237.05]	94 <b>Pu</b> [244.06]	95 <b>Am</b> [243.06]	96 <b>Cm</b> [247.07]	97 <b>Bk</b> [247.07]	98 <b>Cf</b> [251.08]	99 <b>Es</b> [252.08]	100 <b>Fm</b> [257.10]	101 <b>Md</b> [258.10]	102 <b>No</b> [259.10]	103 <b>Lr</b> [262.11]	