

Roger Horn

University of Utah



April 27, 2009

2:00 p.m.

Classroom Building 410 (CL410)

Matrix Canonical Form

You are in your office; the door is open. A well-dressed visitor walks in confidently without knocking. He has a single sheet of wrinkled paper in his left hand and a partially open brown envelope in his right hand. Without any introduction, he strides to your desk and drops the paper and envelope on top of the ungraded midterm exams and unfinished referee reports. Glancing at the paper, you see a bold red "Eyes Only" stamp and a complex 7-by-7 matrix; visible in the envelope is a neat stack of crisp \$100 bills.

"Tell me about that matrix," he says.

Where to begin? "It has rank five," or "It is nilpotent and has Jordan blocks of sizes three and four," or "Its Hermitian part is positive semidefinite," or "It is congruent to a diagonal matrix." Probably not a good way to begin ... better to ask questions such as "Where do the data come from?", "Are there any relevant symmetries or invariants?", and "Does this matrix interact with others in some way?"

The purpose of your questions is to discover whether there is a natural equivalence relation lurking behind your visitor's matrix; if so, it is likely that reducing it to a corresponding canonical form will be illuminating.

We will give examples of a variety of matrix equivalence relations that arise in practice and will discuss canonical forms corresponding to some of them. We will pay special attention to an alternative to the Jordan canonical form for similarity, and to a recently discovered canonical form for congruence.



UNIVERSITY OF
REGINA

Mathematics
and
Statistics



Pacific
Institute
FOR THE
MATHEMATICAL
SCIENCES