UNIVERSITY OF REGINA
Department of Mathematics and Statistics

Math 103 - 001, L01
Final Exam, Fall 2023

Time: 3 hours

Name: _____

Student Number: _____

Instructor (please check):
- ☐ Patrick Maidorn (001)
- ☐ Iqbal Husain (L01)

INSTRUCTIONS

1. Full credit is awarded only for well-presented, correct solutions in which all your work is shown.

2. Your solution and final answer should appear on the right side of the pages in this exam booklet; the left side pages are to be used for scrap paper and are provided for rough work only. If you require more space for your solution than the right side allows, then use the left side (facing) page and indicate on your solution that there is work to be found on the facing page.

3. The marks allocated for each question are indicated.

4. Graphing calculators, tablets, and phones are not permitted.

For instructor use only:

Page:	1	2	3	4	5	6	7	Total
Marks:	15	14	14	17	16	14	10	100
Score:								

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1. Evaluate the following limits. *[9 marks]*

a) $\lim_{x\to 2}\dfrac{x^3+2x^2-8x}{x^2-4}$

b) $\lim_{t\to 0}\dfrac{\sqrt{9+t}-3}{t}$

c) $\lim_{x\to 4^+}\dfrac{5x-4-x^2}{x^2-8x+16}$

2. Using the limit definition of the derivative, find the derivative of $f(x)=\dfrac{x}{x+1}$.

You will not be given marks if you use the differentiation rules instead of the definition.

[6 marks]

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3. Find the indicated derivatives. You do not need to simplify your answers.

[14 marks]

a) $f(x) = \frac{3}{x^2} + \sqrt[4]{x} - e^{-3x} + \ln(x) - 4$ $f'(x) = ?$ (Do not simplify)

b) $f(x) = x^3\sqrt{x^2 + 2x - 1}$ $f'(x) = ?$ (Do not simplify)

c) $g(t) = \frac{1 + e^t}{1 - e^t}$ $g'(t) = ?$ (Simplify final answer)

d) $y(x) = \ln(1 + \ln x)$ $y''(1) = ?$ (Simplify final answer)

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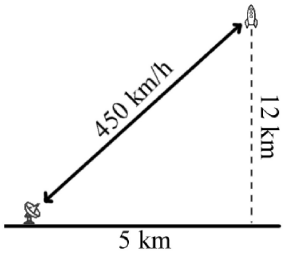
4. Find the equation of the tangent line (in form $y = mx + b$)
to the curve $\sqrt{x} + \sqrt{y} + xy = 7$ at the point $(x, y) = (1,4)$. *[7 marks]*

5. A radar station is tracking the vertical flight of a rocket. The station is 5 km from the rocket's launch site. Currently, the rocket is at a height of 12 km, and the direct distance between rocket and radar station is increasing at a rate of 450 km/h. What is the vertical velocity of the rocket at this time? *[7 marks]*



This page is to be used for work on questions #6 and #7.

6. Find and state the local extreme points and inflection points for the function $f(x) = 2x^3 + 5x^2 - 4x$. Use this information to sketch a graph of the function. Use the left side of this page for rough work, and only record your final answers here. *[10 marks]*

(x,y) values of local minima: (x,y) values of local maxima: (x,y) values of inflection points:	Graph:

7. Find and state all intercepts as well as vertical and horizontal asymptotes of $f(x) = \dfrac{1-x}{x^2-1}$.

Use this information to sketch a graph of the function. Use the left side of this page for rough work, and only record your final answers here. *[7 marks]*

Intercept(s): Vertical Asymptote(s): Horizontal Asymptote(s):	Graph:

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8. A manufacturer has been selling 1500 television sets a week for $540 each. A market survey indicates that for each $12 rebate offered to a buyer, the number of sets sold will increase by 120 per week. How large a rebate (in $) should the company offer to a buyer in order to maximize its revenue? *[10 marks]*

9. The bird population in a wildlife sanctuary is decreasing exponentially. Each year, the population is reduced by 20% (that is, the population level is reduced to 80% of the previous year's level). If there are an estimated 50,000 birds in the sanctuary today, how long will it take for the population to drop below 20,000 birds? *[6 marks]*

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10. Integrate. *[14 marks]*

a) $\int_1^4 \frac{(x+2)^2}{\sqrt{x}} dx$

b) $\int t^{1/2}(t^{3/2}-1)^5 dt$

c) $\int_{-1}^0 \frac{e^x}{e^x+1} dx$

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11. Find the area enclosed by the curves $x + y^2 = 9$ and $y^2 - x = 9$.

For full marks, your solution should include a sketch of the curves with the relevant area shaded and relevant intersection points clearly labeled. *[10 marks]*

The End! Have a great holiday! ☺