# UNIVERSITY OF REGINA
## Department of Mathematics and Statistics

## Math 102 - 001
## Final Exam, Winter 2024

Time: 3 hours

Name: _____

Student Number: _____

### INSTRUCTIONS

1. This exam consists of fifty (50) multiple choice questions, each with answers A-F.

2. In each case, select the one best answer for a question. If you select multiple answers, you will not receive points for the question.

3. No work needs to be shown, you are only graded on the final answer.

4. **IMPORTANT:**
   Clearly **record your answer** on the back page of this exam (the "Answer Sheet").
   This answer sheet is what is graded. It is recommended that your reserve at least five minutes at the end of the exam to double-check that all answers are recorded correctly.

   **Do not detach the answer sheet from the exam.**

5. There are several blank pages throughout this exam. These pages are left intentionally blank and can be used as scratch paper for rough work.

6. Each correct answer is worth 2 points, for a total of 100 points. There are no penalties for incorrectly answered questions.

7. You are allowed to use a standard (non graphing) scientific calculator.
   Laptops, tablets, and phones are not permitted.

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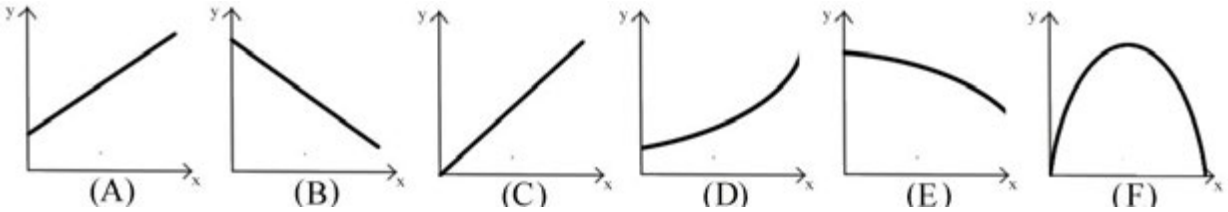
**Note: Questions #1-#4 are based on the following information:**

An object is being cooled at a constant rate. Currently (time t=0) the temperature is 24.5ºC. Every three minutes, the temperature is reduced by 0.6ºC.

1. If we modeled the temperature T (in ºC) of the object as a function of time t (in minutes), which graph shape (A-F) best describes this relationship?




2. What function equation models the temperature T of the object as a function of time t?

(A) $T(t) = 24.5 + 0.6\ t$
(B) $T(t) = 0.6 + 24.5\ t$
(C) $T(t) = 24.5 - 0.6\ t$
(D) $T(t) = 24.5 - 0.2\ t$
(E) $T(t) = 24.5 - 1.8\ t$
(F) $T(t) = 24.5 + 0.2\ t$

3. What is the temperature in five minutes from now?

(A) 24 ºC
(B) 23.8 ºC
(C) 23.5 ºC
(D) 23.1 ºC
(E) 22.7 ºC
(F) 21.5 ºC

4. In how many minutes will the temperature of the object reach 20 ºC?

(A) In 6 minutes from now.
(B) In 7.5 minutes from now.
(C) In 2.5 minutes from now.
(D) In 15 minutes from now.
(E) In 22.5 minutes from now.
(F) In 30.25 minutes from now.

5. Two objects are heated at the same time. Object A's starting temperature (at t=0) is 10.4ºC, and every minute its temperature increases by 1.8ºC. Object B's starting temperature (at t=0) is 15.2ºC, and every minute its temperature increases by 0.4ºC. In how many minutes will the temperature of the two objects be exactly the same? Round to two decimals.

(A) In 3.43 minutes from now.
(B) In 4.80 minutes from now.
(C) In 3.15 minutes from now.
(D) In 2.18 minutes from now.
(E) In 8.45 minutes from now.
(F) In 5.13 minutes from now

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6. If we write the expression $\frac{x^2\sqrt{x}}{(x^3x^{-1})^2}$ as a single exponent $x^n$, then what is the value of *n*?

(A) n=-5/2
(B) n=-3/2
(C) n=-1
(D) n=1/2
(E) n=2
(F) n=7/2

7. Fully factor the expression $x^3 + x^2 - 3x - 3$.

(A) $x(x^2 + x - 3)$
(B) $(x^2 + 3)(x - 1)$
(C) $(x^2 + 1)(x - 3)$
(D) $(x - \sqrt{3})(x + \sqrt{3})(x + 1)$
(E) $(x - 3)(x + 3)(x + 1)$
(F) $(x - 1)(x + 1)(x - \sqrt{3})(x + \sqrt{3})$

8. Simplify the expression $\left(\frac{1}{x+1} - \frac{1}{x}\right) \div \left(\frac{1}{x}\right)$.

(A) 0
(B) $\frac{x}{x+1}$
(C) $\frac{-1}{x+1}$
(D) $x - 1$
(E) $1 - x$
(F) $\frac{1-x}{1+x}$

9. Solve the inequality $-1 < 1 - 3x \leq 3$. Write your answer in interval notation.

(A) (2/3 , 3/2]
(B) (-2/3 , 2/3]
(C) (-2/3 , 3/2]
(D) [-2/3 , 2/3)
(E) (-3/2 , -2/3]
(F) [-3/2 , -2/3)

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10. Solve the equation $x^2 + x = 12$.

(A) x=2 and x=6
(B) x=2 and x=-6
(C) x=3 and x=-4
(D) x=2 and x=-4
(E) x=-3 and x=4
(F) x=-2 and x=6

11. Solve the inequality: $x^2 + 7x + 10 > 0$

(A) x<2 or x>5
(B) x<-2 or x>5
(C) x<-5 or x>-2
(D) x>-5 and x<-2
(E) x<2 or x>5
(F) x>2 and x<5

12. Solve the equation: $|3 - 2x| = 1 - x$

(A) x=2
(B) x=0 and x=2
(C) x=4/3
(D) x=2 and x=4/3
(E) x=0 and x=4/3
(F) This equation has no solutions.

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**Note: Questions #13-#14 are based on the following information:**

A closed rectangular box has a square base. The box is four times as tall as it is wide. It has a volume of exactly 13.5 $m^3$.

13. What is the height of the box?

(A) 6 m
(B) 5.5 m
(C) 3.25 m
(D) 4.25 m
(E) 5.25 m
(F) 5.125 m

14. What is the surface area of the box?

(A) 40.5 $m^2$
(B) 35.25 $m^2$
(C) 10.5 $m^2$
(D) 13.5 $m^2$
(E) 32 $m^2$
(F) 37.25 $m^2$

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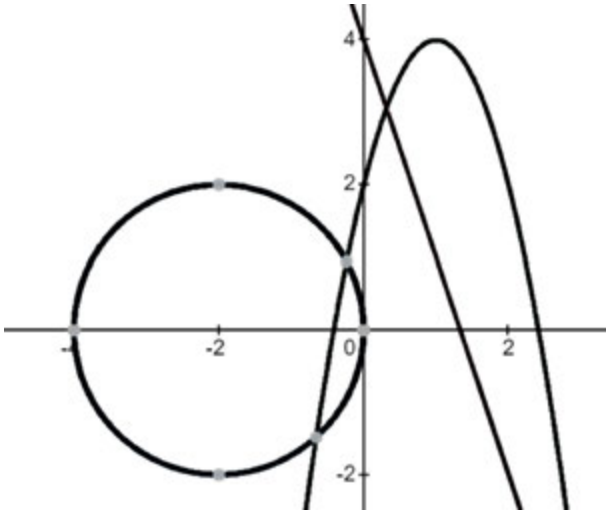
**Note: Questions #15-#19 are based on the following information:**

The given picture shows the graph of a line, a circle, and a parabola:




15. What is the equation of the line?
Note that it travels through points (0,4) and (2,-2).

(A) $y=4+3x$
(B) $y=4-3x$
(C) $y=4-x$
(D) $y=3-4x$
(E) $y=4-2x$
(F) None of the above.

16. What is the equation of the circle?
Note that it has its centre at (-2,0).

(A) $(x-2)^2 + y^2 = 2$
(B) $(x+2)^2 + (y-2)^2 = 2$
(C) $(x-2)^2 + y^2 = 4$
(D) $(x+2)^2 + y^2 = 2$
(E) $(x+2)^2 + y^2 = 4$
(F) None of the above.

17. What is the equation of the parabola? Note that it has its vertex at x=1.

(A) $y=4-x^2$
(B) $y=4-(x-1)^2$
(C) $y=2-4x-2x^2$
(D) $y=2+4x-2x^2$
(E) $y=4-(x+1)^2$.
(F) None of the above.

18. What is the circumference of the circle?

(A) $2ð$
(B) $4ð$
(C) $4ð^2$
(D) $3ð/4$
(E) $ð$
(F) $ð^2$

19. The line and the parabola will intersect each other twice. One intersection point is shown in the picture above, at a height of *approximately* y=3. The other intersection point is not shown in the picture. What is its y-value, rounded to two decimal places.

(A) $y=-5.48$
(B) $y=-5.52$
(C) $y=-5.56$
(D) $y=-5.61$
(E) $y=-5.65$
(F) $y=-5.68$

**Note: Questions #20-#23 are based on the following information:**

Suppose a baker is selling pretzels at a local farmers' market. Let x be the number of pretzels sold each day. Right now, they are selling 120 pretzels per day at a price of $2.25 each. Increasing the price to $2.50 each reduces the number of pretzels sold to 110 per day. Assume that the price function p (i.e. the price at which x pretzels could be sold) is linear and has form $p = mx + b$.

20. How many pretzels could be sold at a price of $3.00 each?
(A) 90
(B) 125
(C) 75
(D) 150
(E) 130
(F) 100

21. What is the slope of the price function, i.e. the value of $m$?
(A) m=40
(B) m=-40
(C) m=1/40
(D) m=-1/40
(E) m=-1/30
(F) m=30

22. What is the equation of the price function?
(A) $p(x) = -\frac{x}{30} + 6.25$
(B) $p(x) = -40x + 5.5$
(C) $p(x) = \frac{x}{40} - 0.5$
(D) $p(x) = 30x + 6.5$
(E) $p(x) = -\frac{x}{40} + 5.25$
(F) $p(x) = -\frac{x}{30} + 3.75$

23. Suppose the farmers' market charges a $50 fee to rent the place for the day, and each pretzel costs the baker $0.75 to make. What is the total profit/loss for the day if they plan to make and sell 150 pretzels?
(A) A profit of $125
(B) A loss of $62.50
(C) A loss of $162.50
(D) A profit of $162.50
(E) A profit of $62.50
(F) A profit of $175

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**Note: Questions #24-#27 are based on the following information:**

The height of a toy drone (in metres, after t seconds) is given by the function $h(t) = -4t^2 + 44t + 79$ .

24. What is the height of the drone after two seconds?
(A) 77 m
(B) 175 m
(C) 250 m
(D) 151 m
(E) 75 m
(F) 126 m

25. After how many seconds will the drone hit the ground (i.e. reach height zero)? Round to two decimals.
(A) 14.42 seconds
(B) 12 seconds
(C) 14.46 seconds
(D) 13.52 seconds
(E) 11 seconds
(F) 12.57 seconds

26. What is the highest point (in metres) of the projectile's trajectory?
(A) 205 m
(B) 200 m
(C) 191 m
(D) 199 m
(E) 210 m
(F) 192 m

27. At what time will the projectile reach its highest point?
(A) In 6 seconds from now.
(B) In 6.5 seconds from now.
(C) In 5.5 seconds from now.
(D) In 7 seconds from now.
(E) In 7.5 seconds from now.
(F) In 8 seconds from now.

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28. Which of the following statements is FALSE?
(A) The graph of a quadratic equation is called a parabola.
(B) The form $y = A(x + B)^2 + C$ gives the position of the vertex at $(x, y) = (B, C)$
(C) If a quadratic equation has two x-intercepts, the position of the vertex is halfway between them.
(D) If a quadratic equation has one x-intercept, the position of the vertex is at that intercept.
(E) The sign of "a" in $y = ax^2 + bx + c$ tells you if the parabola opens up (if a>0) or down (if a<0)
(F) None of the above, i.e. ALL of the above statements are TRUE.

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29. Find and state the domain of the function $f(x) = \dfrac{x-2}{x^2-3x}$.

(A) All x except x=0, x=2, x=3.
(B) All $x \geq 2$.
(C) All x in the interval (0,3).
(D) All x except x=0 and x=3.
(E) All $x<0$ and $x>3$.
(F) All x in the interval (2,3).

30. What best describes the symmetry of the function $f(x) = \dfrac{-x}{x^4+3}$:

(A) This function is odd.
(B) This function is neither even nor odd.
(C) This function is even.
(D) This function is both even and odd.
(E) This function is symmetric across the x-axis.
(F) It is impossible to determine the symmetry of this function.

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**Note: Questions #31-#33 are based on the following function:** $f(x) = x^2 - 2x - 3$

31. Find and state all intercepts of the function $f(x)$.

(A) x-intercepts (0,3) and (0,-1), y-intercept (-3,0).
(B) x-intercept x=2, y-intercept y=-3
(C) x-intercept x=-3, y-intercept y=2
(D) x-intercepts (3,0) and (-1,0), y-intercept (0,-3)
(E) x-intercepts (-3,0) and (1,0), y-intercept (0,-3)
(F) x-intercepts (-3,0) and (1,0), y-intercept (0,3)

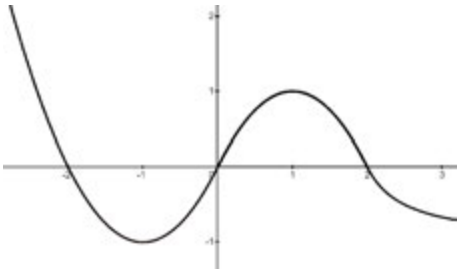
32. Given the function $f(x) = x^2 - 2x - 3$, find an expression for $f(x-1)$.

(A) $(x-3)(x+1)$
(B) $x^2 - 2x - 4$
(C) $x^2 - 4x - 2$
(D) $4x^2 - 4x + 1$
(E) $(x+3)(x-1)$
(F) $x(x-4)$

33. Given the function $f(x) = x^2 - 2x - 3$, find an expression for $f(x-1) - f(x) - 1$.

(A) $2 - 2x$
(B) $-x^2 - 1$
(C) $-2x$
(D) $x - x^2$
(E) $-1$
(F) $x^2 - 1$

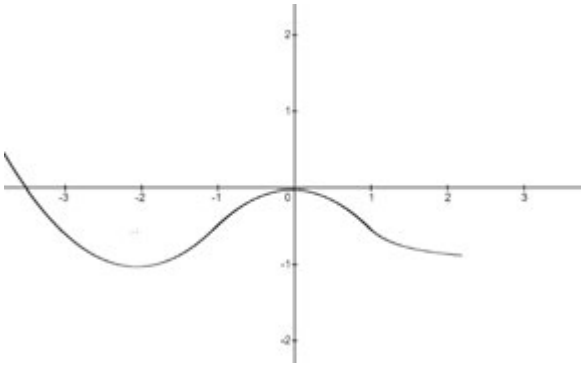
34. Consider the given graph of the function $y = f(x)$:



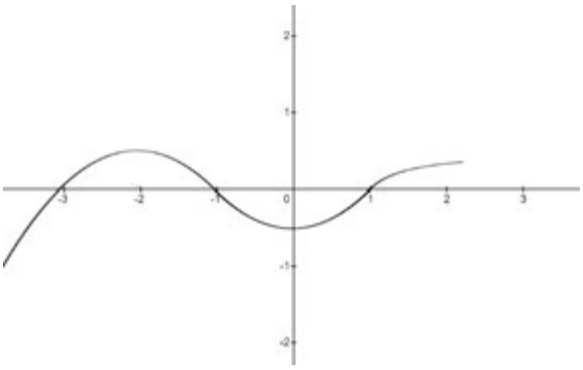
Which of the following graphs best represents the graph of

$$y = \frac{1}{2} f(-x) + \frac{1}{2}$$

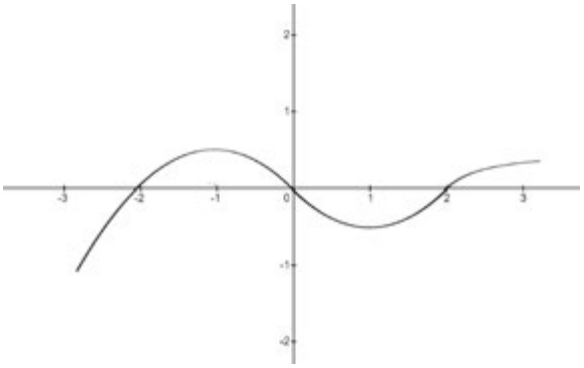
(A)



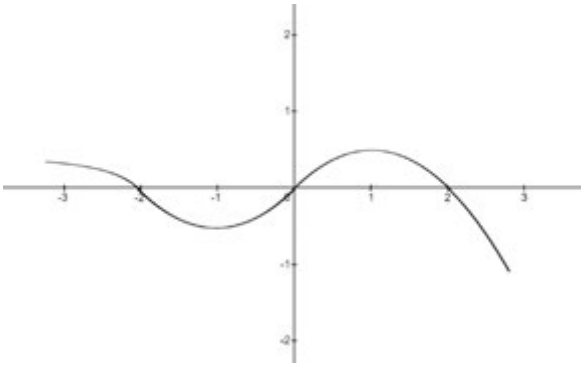
(B)



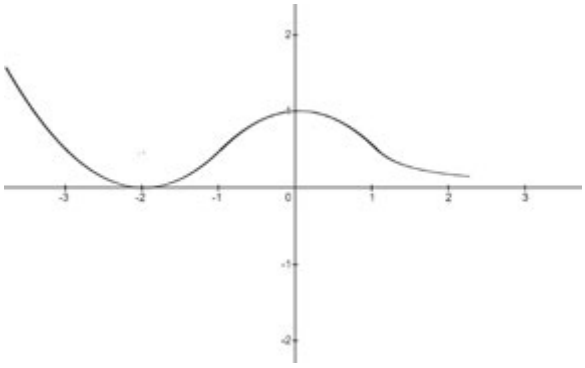
(C)



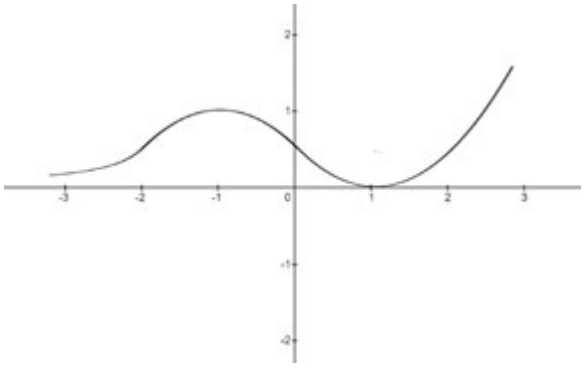
(D)



(E)



(F)



35. If the graph of $y = f(x)$ contains the point (x,y)=(5,-1), then the graph of $y = \frac{1}{2} f(-x) + \frac{1}{2}$ ...

(A) ... contains the point (x,y)=(-5/2, -1/2)
(B) ... contains the point (x,y)=(-5, 0)
(C) ... contains the point (x,y)=(-5/2, 0)
(D) ... contains the point (x,y)=(-5, -1/4)
(E) ... contains the point (x,y)=(5/2, -1/4)
(F) ... contains the point (x,y)=(5/2, 0)

36. What is the degree and leading coefficient of the polynomial $p(x) = (2-x)^2(3x+1)(3x-1)$?

(A) degree = 2 , leading coefficient = 2
(B) degree = 4 , leading coefficient = 3
(C) degree = 2 , leading coefficient = -3
(D) degree = 4 , leading coefficient = 9
(E) degree = 4 , leading coefficient = -9
(F) degree = 2 , leading coefficient = 3

37. Write as a single logarithm: $3\log_2(x) - \log_2(x+1) + 4$

(A) $\log_2\left(x^3 - x + 3\right)$

(B) $\log_2\left(\frac{2^4 x^3}{x+1}\right)$

(C) $\log_2\left(\frac{3x+2}{x+1}\right)$

(D) $\log_2\left(\frac{2x^3}{x+1}\right)$

(E) $\log_2\left(x^3 - x + 1\right)$

(F) $\log_2\left(\frac{16x^3}{x+1}\right)$

38. Solve the equation $5 - 3e^{3x+1} = 2$

(A) $x = -\frac{1}{3}$

(B) $x = \ln\left(\frac{5}{3}\right)$

(C) $x = \frac{\ln(5)}{3}$

(D) $x = \ln(3) - 5$

(E) $x = 3$

(F) $x = \ln(3)$

39. Find the exact value of $\log_5(0.04)$

(A) -0.2
(B) $5^{-0.04}$
(C) -0.008
(D) -2
(E) ln(5)
(F) This value is not defined.

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40. A $10,000 investment is made into an account earning 6.25% compounded monthly. If it is invested for 5 years, what is the value of the investment after that time?

(A) $13,657.30
(B) $53,216.09
(C) $13,540.81
(D) $13,125.00
(E) $48,199.32
(F) $14,104.47

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**Note: Questions #41-#43 are based on the following information:**

A radioactive material has a half-life of 27.3 minutes. The amount *A* after *t* minutes will be modeled by the continuous exponential decay model $A(t) = A_0 e^{kt}$.

41. If we start with 100g of the material, how much will remain after 3×27.3=81.9 minutes?
(A) 18.1 g
(B) 81.9 g
(C) 54.6 g
(D) 12.5 g
(E) 25.0 g
(F) 10.4 g

42. What is the value of k, rounded to five decimals?
(A) -0.02539
(B) 0.03663
(C) -0.03663
(D) 0.02539
(E) -0.01103
(F) 0.01103

43. After how many minutes will only 1g of the original 100g remain?
(A) 78.77 minutes
(B) 181.38 minutes
(C) 102.87 minutes
(D) 207.37 minutes
(E) 185.54 minutes
(F) 111.12 minutes

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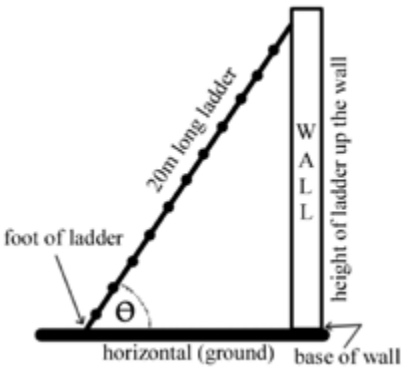
44. Convert 10º into radian measure.
(A) ð/10
(B) 2ð/9
(C) 9ð/2
(D) 18ð
(E) ð/18
(F) 9ð/10

45. A 20m long ladder is leaning against a vertical wall. The angle that the foot of the ladder makes with the horizontal is currently è=78º. How high up the wall will the top of the ladder reach?




(A) 19.56m
(B) 4.16m
(C) 9.41m
(D) 11.98m
(E) 10.23m
(F) 17.16m

46. At what angle è (with the horizontal, in radians) would you need to lean a 20m long ladder against a vertical wall, if you want the foot of the ladder to be exactly 10m away from the base of the wall?

(A) ð/9
(B) ð/6
(C) ð/4
(D) ð/3
(E) ð/2
(F) ð

47. If sin(è)=3/5 and è is an angle in the second quadrant, what is the value of cos(è)?

(A) 4/5
(B) 1/2
(C) 2/5
(D) -2/5
(E) -4/5
(F) -1/2

48. Find all solutions to $0 = 2\sin(x) + 1$ in the interval [0,4ð]

(A) ð/6, 5ð/6, 7ð/6, 11ð/6
(B) 5ð/6, 7ð/6, 17ð/6, 19ð/6
(C) 7ð/6, 11ð/6, 17ð/6, 19ð/6
(D) 7ð/6, 11ð/6, 19ð/6, 23ð/6
(E) ð/6, 5ð/6, 13ð/6, 17ð/6
(F) ð/6, 11ð/6, 13ð/6, 23ð/6

**Note: Questions #49-#50 are based on the following information:**

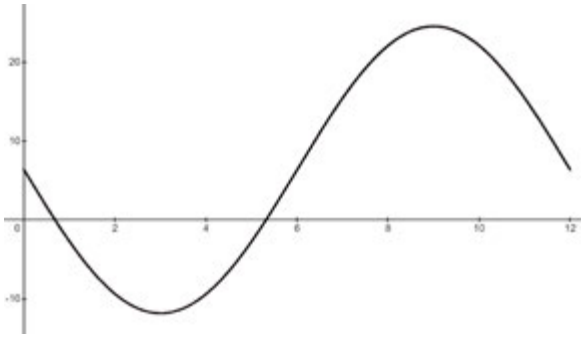
The yearly average temperature (in ºC) in MathTown is given by $T(t) = 6.4 - 18.2\sin(t\pi/6)$, where t is in months ($0 \le t \le 12$, t=0 is January 1st) and is in radian measure.

49. What is the approximate temperature in MathTown on February 15th (i.e. t=1.5)?

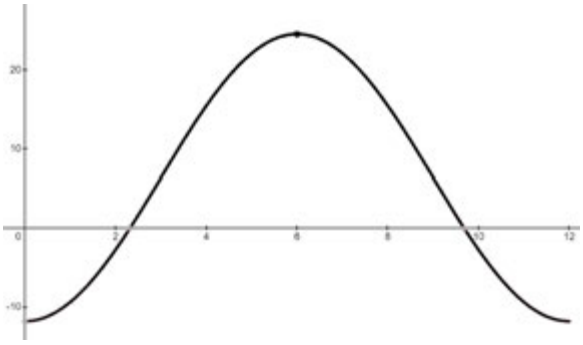
(A) -11.8ºC
(B) -8.23ºC
(C) -6.47ºC
(D) 1.8ºC
(E) 5.32ºC
(F) 10.21ºC

50. Which of the following graphs best represents the function T(t)?

(A)



(B)



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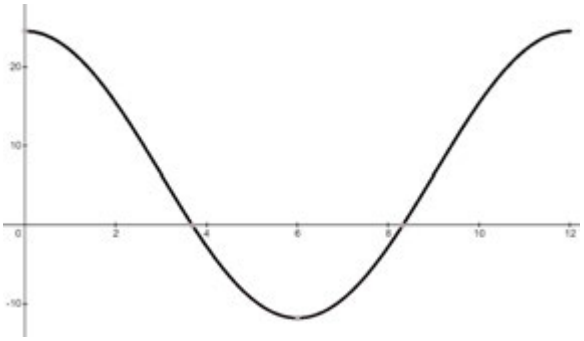

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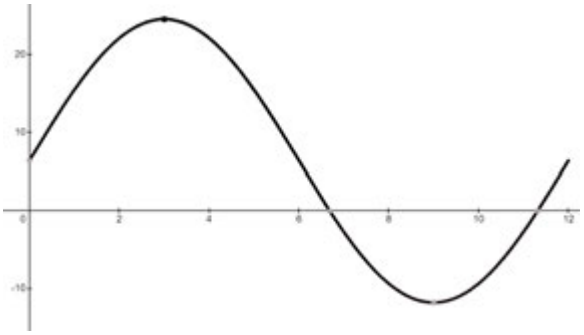

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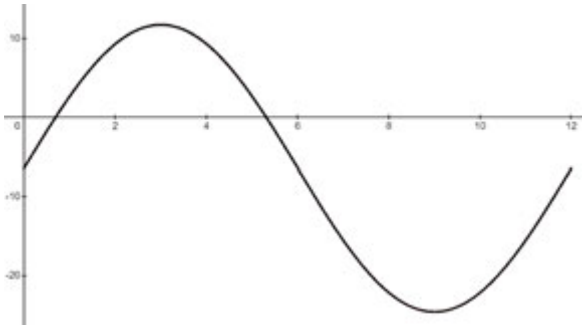
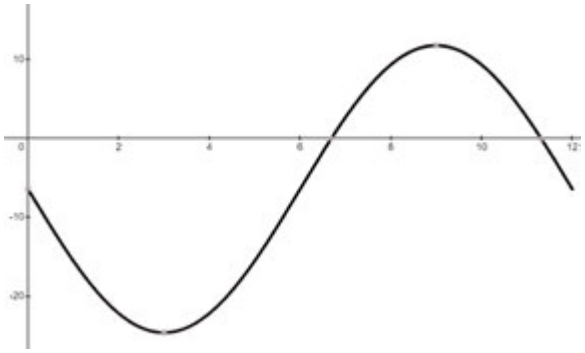


(F)



## Answer Sheet - Record all answers here

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | A | B | C | D | E | F | 26 | A | B | C | D | E | F |
| 2 | A | B | C | D | E | F | 27 | A | B | C | D | E | F |
| 3 | A | B | C | D | E | F | 28 | A | B | C | D | E | F |
| 4 | A | B | C | D | E | F | 29 | A | B | C | D | E | F |
| 5 | A | B | C | D | E | F | 30 | A | B | C | D | E | F |
| 6 | A | B | C | D | E | F | 31 | A | B | C | D | E | F |
| 7 | A | B | C | D | E | F | 32 | A | B | C | D | E | F |
| 8 | A | B | C | D | E | F | 33 | A | B | C | D | E | F |
| 9 | A | B | C | D | E | F | 34 | A | B | C | D | E | F |
| 10 | A | B | C | D | E | F | 35 | A | B | C | D | E | F |
| 11 | A | B | C | D | E | F | 36 | A | B | C | D | E | F |
| 12 | A | B | C | D | E | F | 37 | A | B | C | D | E | F |
| 13 | A | B | C | D | E | F | 38 | A | B | C | D | E | F |
| 14 | A | B | C | D | E | F | 39 | A | B | C | D | E | F |
| 15 | A | B | C | D | E | F | 40 | A | B | C | D | E | F |
| 16 | A | B | C | D | E | F | 41 | A | B | C | D | E | F |
| 17 | A | B | C | D | E | F | 42 | A | B | C | D | E | F |
| 18 | A | B | C | D | E | F | 43 | A | B | C | D | E | F |
| 19 | A | B | C | D | E | F | 44 | A | B | C | D | E | F |
| 20 | A | B | C | D | E | F | 45 | A | B | C | D | E | F |
| 21 | A | B | C | D | E | F | 46 | A | B | C | D | E | F |
| 22 | A | B | C | D | E | F | 47 | A | B | C | D | E | F |
| 23 | A | B | C | D | E | F | 48 | A | B | C | D | E | F |
| 24 | A | B | C | D | E | F | 49 | A | B | C | D | E | F |
| 25 | A | B | C | D | E | F | 50 | A | B | C | D | E | F |