UNIVERSITY OF REGINA
Department of Mathematics and Statistics

Math 102 - 001, 002
Final Exam, Fall 2023

Time: 3 hours　　　　　Name: _____

Please indicate your section:　　Student Number: _____
☐ 001 (Maidorn)
☐ 002 (Osmanli)

INSTRUCTIONS

1.　Full credit is awarded only for well-presented, correct solutions in which all your work is shown, unless otherwise indicated.

2.　Your solution and final answer should appear on the right side of the pages in this exam booklet; the left side pages are to be used for scrap paper and are provided for rough work only. If you require more space for your solution than the right side allows, then use the left side (facing) page and indicate on your solution that there is work to be found on the facing page.

3.　The marks allocated for each page are indicated below.

4.　Laptops, tablets, and phones are not permitted.

For instructor use only:

Page:	1	2	3	4	5	6	7	Total
Marks:	14	16	16	14	12	14	14	100
Score:								

1. Record your answers in the "Answer" boxes. Each question is worth 2 marks.
   Use the left side of this page for any rough work (however, only the final answer is graded).
   [14 marks]

a) Simplify the expression by writing it as a single exponential:
$$\frac{x^2 x^3}{\sqrt{x}}$$

Answer

b) Solve the inequality $-1 < 3x - 2 \le 1$.
   Write your final answer in interval notation.

Answer

c) Solve the equation $4x^2 + 3x = 1$.

Answer

d) Convert $40°$ to radian measure. Write your answer as a fraction in lowest terms.

Answer

e) Find and state the value of $\tan\left(\dfrac{15\pi}{4}\right)$

Answer

f) Find and state the value of $\log_4\left(\dfrac{1}{64}\right)$

Answer

g) Simplify the expression by writing it as a single logarithm:
$$\log_3(x+1) - 3\log_3(x)$$

Answer

This page is to be used as scrap paper.

2. Record your answers on the "Answer" line. Each question is worth 4 marks.
   A correct answer in the space provided is given full marks regardless of work shown. However you can earn partial marks for an incorrect answer by showing some correct work in the space provided.   *[16 marks]*

a) Fully factor the expression $2x^6 + x^5 - 2x^4 - x^3$

Final Answer: _____

b) Solve the equation $1 + \dfrac{1}{x-1} = \dfrac{6}{x^2 - 1}$.

Final Answer: _____

c) Find all x-values in the interval $[0, 4\pi]$ such that $2\sin(x) + 3 = 2$.

Final Answer: _____

d) Solve $|1 - x| < \dfrac{x}{2}$. Write your final answer in interval notation.

Final Answer: _____

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3. Consider the function $f(x) = \dfrac{x}{4 - x^2}$. [16 marks]

a) Find and state its domain. Answer: _____

b) Is the function even, odd, or neither? Answer: _____

c) Evaluate and simplify the expression for $f(1 - 3x)$.

d) In your own words, describe the transformations (shifting/scaling) in the correct order that would produce the graph of $y = 2f(-x) + 1$ given the original graph $y = f(x)$.

e) The point with coordinates $(x, y) = (1, \tfrac{1}{3})$ is a point on the graph of $y = f(x)$. What are the coordinates of the corresponding point on the graph of $y = 2f(-x) + 1$?

Answer: _____

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4. In its first year, the weight of a terrier puppy increases at a constant rate. At age ten weeks the puppy weighs 6.2 kg and at age 26 weeks the puppy weighs 19.7 kg.   *[8 marks]*

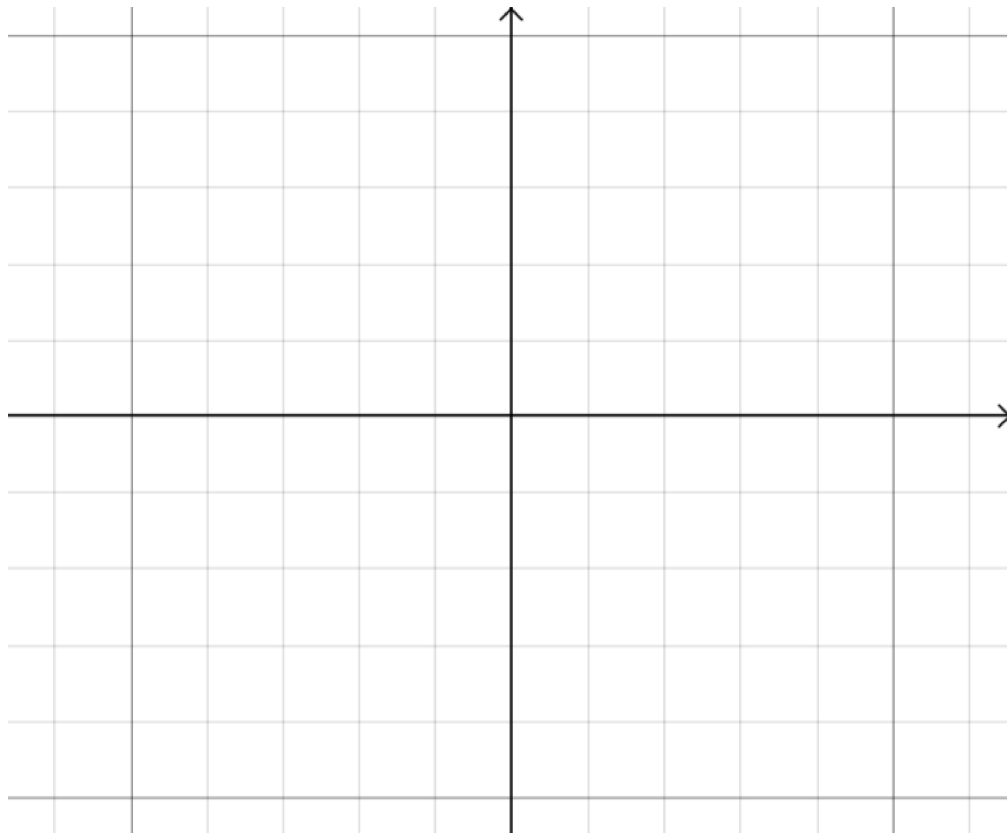a) Find and state the weight of the puppy (W, in kg) as a function of the its age (t, in weeks).

b) What does the model predict the puppy will weigh at age 52 weeks? Round to one decimal.

5. Sketch the graphs of the following two equations on the given coordinate system:
                       [6 marks]

(i) $y = 3 - 2x$ (ii) $(x + 2)^2 = 9 - y^2$

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6. Two planes are passing by each other. The direct distance between the two planes (in km) as a function of time (in minutes) is given by the equation $D(t) = t^2 - 11t + 30.75$.

[12 marks]

a)   How far apart are the planes   at t=0 minutes?   _____

                                    at t=2 minutes?   _____

b)   At what time will the two planes be closest to each other? State both the time as well as their distance from each other at that time.

c)   Sketch a careful graph of the function over a ten minute time period.

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7. The size of an animal population is decaying exponentially. Every five years, the population loses 25% of its members (that is, 75% of the population remain after five years). After what time will the population size be exactly half of its current size? Round your answer to two decimals.   *[8 marks]*

8. Consider the polynomial $y = x^3(1-x)(2x+1)^2$

State the degree: ___n=_____   State the leading coefficient: __a_n=_____

Sketch a graph of the polynomial. Your graph should correctly show all roots, but does not need to include specific y-values.   *[6 marks]*

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9. A 25 metre long rope is attached to the top of a vertical pole and anchored to a spot on the ground some distance away. Assume the rope is tightly drawn, that is we can model it as a perfectly straight line without any sagging. If the angle the rope makes with the horizontal is 32°, how tall is the pole and what is the distance between the pole and the anchor spot on the ground?   *[7 marks]*

10. Moss is growing on the forest floor in a perfectly circular area. Right now, the area of the moss circle is 40 m². Over the next four months, the radius of the circle increases by 5 metres. What is the area of the moss circle now?   *[7 marks]*

The end! Have a great holiday!