Math 217

Assignment 1

Solution Manual

Problem 1. Second order – linear

Problem 2. Third order – non-linear

Problem 3. Second order – non-linear

Problem 4.

(a) $y' = 3e^{3x}\cos 2x - 2e^{3x}\sin 2x$

$\rightarrow y'' = 9e^{3x}\cos 2x - 6e^{3x}\sin 2x - 6e^{3x}\sin 2x - 4e^{3x}\cos 2x$

Thus

$$y'' - 6y' + 13y = (9e^{3x}\cos 2x - 6e^{3x}\sin 2x - 6e^{3x}\sin 2x - 4e^{3x}\cos 2x)$$
$$-6(3e^{3x}\cos 2x - 2e^{3x}\sin 2x)$$
$$+13(e^{3x}\cos 2x)$$
$$= 0 \checkmark$$

(b) $y' = 2c_1 e^{2x} + c_2 e^{2x} + 2c_2 x e^{2x}$

$\rightarrow y'' = 4c_1 e^{2x} + 2c_2 e^{2x} + 2c_2 e^{2x} + 4c_2 x e^{2x}$

Thus

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = \left(4c_1 e^{2x} + 2c_2 e^{2x} + 2c_2 e^{2x} + 4c_2 x e^{2x}\right)$$
$$-4\left(2c_1 e^{2x} + c_2 e^{2x} + 2c_2 x e^{2x}\right)$$
$$+4\left(c_1 e^{2x} + c_2 x e^{2x}\right)$$
$$= 0 \checkmark$$

(c) Using the first part of the "fundamental Theorem of calculus", we have

$$y' = \left(-2x e^{x^2} \int_0^x e^{t^2} dt + e^{-x^2} \cdot e^{x^2}\right) - 2c_1 x e^{-x^2}$$
$$= -2x e^{x^2} \int_0^x e^{t^2} dt + 1 - 2c_1 x e^{-x^2}$$

Thus

$$\frac{dy}{dx} + 2xy = -2x e^{x^2} \int_0^x e^{t^2} dt + 1 - 2c_1 x e^{-x^2}$$
$$+ 2x e^{x^2} \int_0^x e^{t^2} dt + 2c_1 x e^{-x^2} = 1 \checkmark$$

Problem 5.

$$\frac{dy}{dx} + 2xy^2 = 0 \rightarrow \frac{dy}{dx} = -2xy^2 \rightarrow \frac{dy}{y^2} = -2x\,dx$$

$$\rightarrow \int \frac{dy}{y^2} = -2\int x\,dx \rightarrow -y^{-1} = -x^2 + c'$$

$$\rightarrow \boxed{y = 1/(x^2 + c)}$$

Problem 6.

$$e^x y \frac{dy}{dx} = e^{-y} + e^{-2x-y} \rightarrow y\frac{dy}{dx} = e^{-y} \cdot e^{-x} + e^{-2x-y} \cdot e^{-x}$$

$$\rightarrow y\frac{dy}{dx} = e^{-y}(e^{-x} + e^{-3x}) \rightarrow ye^y \frac{dy}{dx} = e^{-x} + e^{-3x}$$

$$\rightarrow \int ye^y\,dy = \int (e^{-x} + e^{-3x})\,dx$$

$$\rightarrow ye^y - y = -e^{-x} - \frac{1}{3}e^{-3x} + c'$$

$$\rightarrow \boxed{y - ye^y = e^{-x} + \frac{1}{3}e^{-3x} + c}$$

Either one is acceptable.

Problem 7.

$$y \ln x \frac{dx}{dy} = \left(\frac{y+1}{x}\right)^2 \rightarrow y \ln x \frac{dx}{dy} = \frac{(y+1)^2}{x^2}$$

$$\rightarrow x^2 . \ln x . dx = \frac{(y+1)^2}{y} dy$$

$$\rightarrow \int x^2 . \ln x . dx = \int \left(y + 2 + \frac{1}{y}\right) dy$$

$\downarrow$ by parts

$$\boxed{\frac{x^3}{3} \ln x - \frac{x^3}{9} + C = \frac{y^2}{2} + 2y + \ln|y|}$$

Problem 8.

$$\sin 3x \, dx + 2y \cos^3 3x \, dy = 0$$

$$\rightarrow \frac{\sin 3x}{\cos^3 3x} dx = -2y \, dy$$

$$\rightarrow 2y \, dy = -\tan 3x . \sec^2 3x \, dx$$

$$\rightarrow \int 2y \, dy = -\int \tan 3x \sec^2 3x \, dx$$

$$\rightarrow \boxed{y^2 = -\frac{1}{6} \tan^2 3x + C}$$

Problem 9.

$$\frac{dx}{dt} = 4(x^2+1) \rightarrow \frac{dx}{x^2+1} = 4\,dt$$

$$\rightarrow \int \frac{dx}{x^2+1} = \int 4\,dt \rightarrow \tan^{-1}(x) = 4t + c;$$

$$x\left(\frac{\pi}{4}\right) = 1 \longrightarrow \tan^{-1}(1) = 4\cdot\frac{\pi}{4} + c$$

$$\rightarrow \frac{\pi}{4} = \pi + c \rightarrow c = -\frac{3\pi}{4}$$

Hence

$$\tan^{-1}(x) = 4t - \frac{3\pi}{4}$$

or

$$\boxed{x = \tan\left(4t - \frac{3\pi}{4}\right)}$$

Either one is acceptable.

Problem 10.

$$(1+x^4)\,dy + x(1+4y^2)\,dx = 0$$

$$\rightarrow \frac{dy}{1+4y^2} = \frac{-x\,dx}{1+x^4} \rightarrow \int \frac{dy}{1+4y^2} = \int \frac{-x\,dx}{1+x^4}$$

$$\rightarrow \frac{1}{2}\tan^{-1}(2y) = \frac{-1}{2}\tan^{-1}(x^2) + c'$$

$$\rightarrow \tan^{-1}(2y) = \tan^{-1}(-x^2) + c$$

$$\rightarrow 2y = \tan\left(\tan^{-1}(-x^2) + c\right)$$

$$\rightarrow y = \frac{1}{2}\tan\left(\tan^{-1}(-x^2) + c\right);$$

$$y(1) = 0 \rightarrow 0 = \frac{1}{2}\tan\left(\tan^{-1}(-1) + c\right)$$

$$\rightarrow \tan\left(-\frac{\pi}{4} + c\right) = 0$$

$$\rightarrow -\frac{\pi}{4} + c = 0$$

$$\rightarrow c = \frac{\pi}{4}$$

$$\rightarrow \boxed{y = \frac{1}{2}\tan\left(\tan^{-1}(-x^2) + \frac{\pi}{4}\right)}$$