UNIVERSITY OF REGINA
DEPARTMENT OF MATHEMATICS & STATISTICS
Mathematics 217-040
Final Exam
Spring 2013

Time: 3 Hours
Instructor: **B. Ahmadi**

NAME: _____
STUDENT NO.: _____

MARKS

5 1. Show the following ODE is exact; then solve it.

$$(x^3 + y^3)dx + 3xy^2 = 0.$$

5 2. Find the general solution of the following ODE.

$$y^{(4)} - 2y'' + y = 0.$$

5 3. **(a)** Solve the following Cauchy-Eular equation.

$$x^2y'' - xy' + y = 0.$$

10 **(b)** Use part (a) and the method of "variation of parameters" to solve the given initial-value problem.

$$x^2y'' - xy' + y = 2x, \qquad y(1) = 0 \quad \text{and} \quad y'(1) = 1.$$

10 4. Use the method of "undetermined coefficients" to solve the following ODE.

$$y'' - 16y = 2e^{4x}.$$

10 5. Use the substitution $u = y'$ to solve the following non-linear ODE.

$$y'' + 2y(y')^3 = 0.$$

5 6. Find the radius of convergence and the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-3)^n}{n\sqrt{n}} x^n.$$

7. Solve the following problems using the MacLaurin series table.

5 **(a)** Find the power series representation of the function

$$f(x) = \frac{\tan^{-1}(x^2)}{x^2}.$$

5 **(b)** Find the sum of the following series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots.$$

10 8. Find the general solution of the following ODE using series.

$$(x-1)y'' + y' = 0.$$

10 9. Find the following Laplace and inverse Laplace transforms (not necessarily using the definition).

$$\mathcal{L}\left[\sin^2(t)\right] =$$

$$\mathcal{L}^{-1}\left[\frac{e^{-\pi s}}{s^2+1}\right] =$$

10 10. Solve the following IVP problem using the Laplace transforms.

$$y'' - 5y' + 6y = \mathcal{U}(t-1); \qquad y(0) = 0, y'(0) = 1.$$

10 11. Find the general solution of the following system of linear ODEs.

$$\mathbf{X}' = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \mathbf{X}$$

Bonus Problem (5 marks). Find the sum of the following series.

$$\frac{\pi}{4} - \frac{\pi^3}{4^3\,3!} + \frac{\pi^5}{4^5\,5!} - \frac{\pi^7}{4^7\,7!} + \cdots.$$

scrap paper:

Good luck!