Math 217, Spring 2013
Assignment 7

1. Find the MacLaurin series of $f(x)$ using the definition of a MacLaurin series (assume that f has a power series representation). Also the radius of convergence.

   (a) $f(x) = \ln(1 + x)$

   (b) $f(x) = e^{-2x}$

   (c) $f(x) = \sinh x$

   (d) $f(x) = \cosh x$

2. Use the MacLaurin series of well-known functions (the table) to find the MacLaurin series of the following finctions.

   (a) $f(x) = \sin(\pi x)$

   (b) $f(x) = e^x + e^{2x}$

   (c) $f(x) = x^2 \ln(1 + x^3)$

   (d) $f(x) = \sin^2 x$ (**Hint.** Use the identity $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$.)

   (e) $f(x) = \tan^{-1}(x^3)$

   (f) $f(x) = e^x \cos x$ (**Hint.** Multiply the the two series term by term.)

3. Use series to evaluate the following limits.

   (a)
   $$\lim_{x \to 0} \frac{x - \ln(1 + x)}{x^2}$$

   (b)
   $$\lim_{x \to 0} \frac{1 - \cos x}{1 + x - e^x}$$

   (c)
   $$\lim_{x \to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$$

4. Find the sums of the series.

(a)

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{3^n}{n5^n}$$

(b)

$$\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n}}{6^{2n}(2n)!}$$

(c)

$$1-\ln 2+\frac{(\ln 2^2)}{2!}-\frac{(\ln 2)^3}{3!}+\cdots$$

(d)

$$\frac{1}{1\cdot 2}-\frac{1}{3\cdot 2^3}+\frac{1}{5\cdot 2^5}-\frac{1}{7\cdot 2^7}+\cdots$$

5. §6.2, Problems 7-14;

6. §6.2, Problem 23;