Using Latin Squares to Test Video Games

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Department of Mathematics and Statistics
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A Dream Job

You are a video game tester and you have to test a retro version of PacMan.

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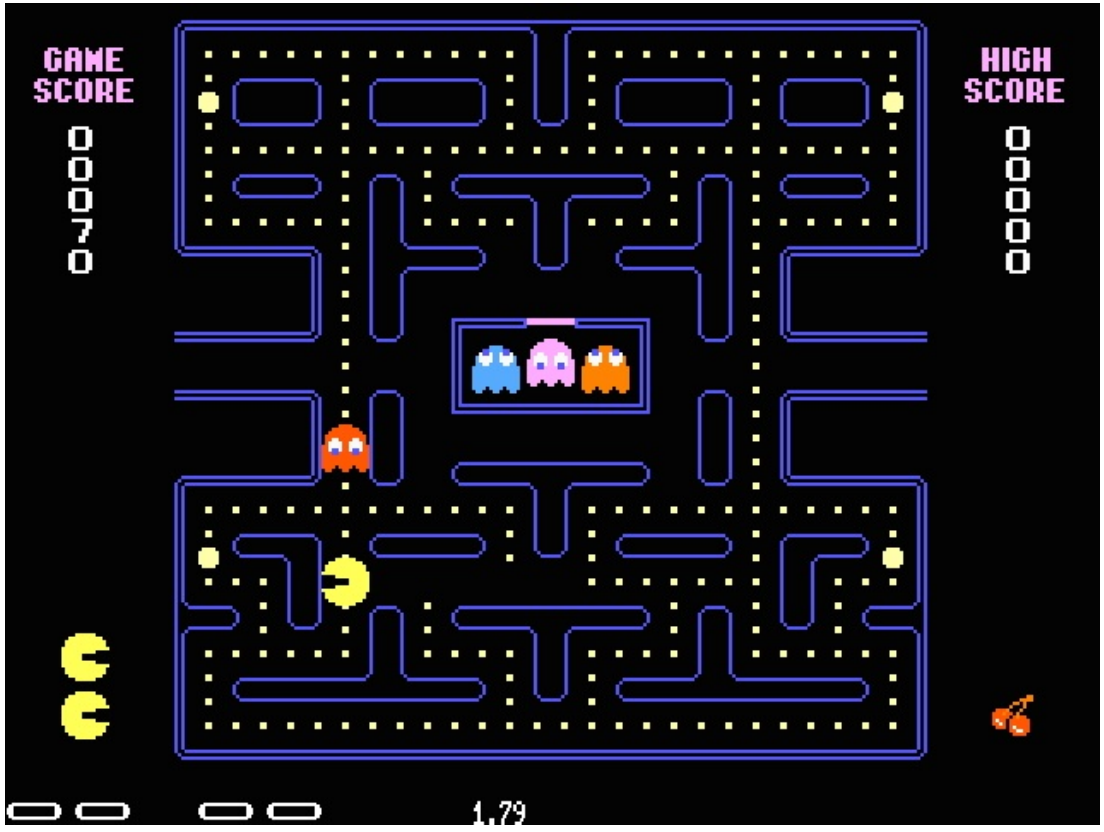

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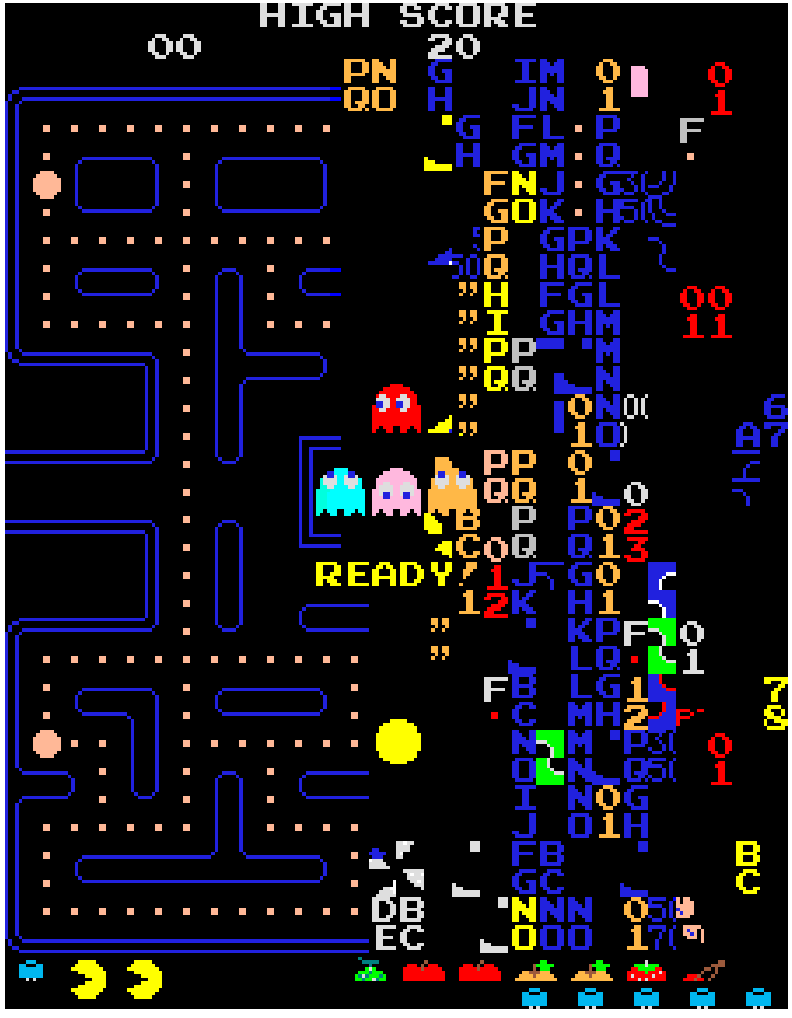

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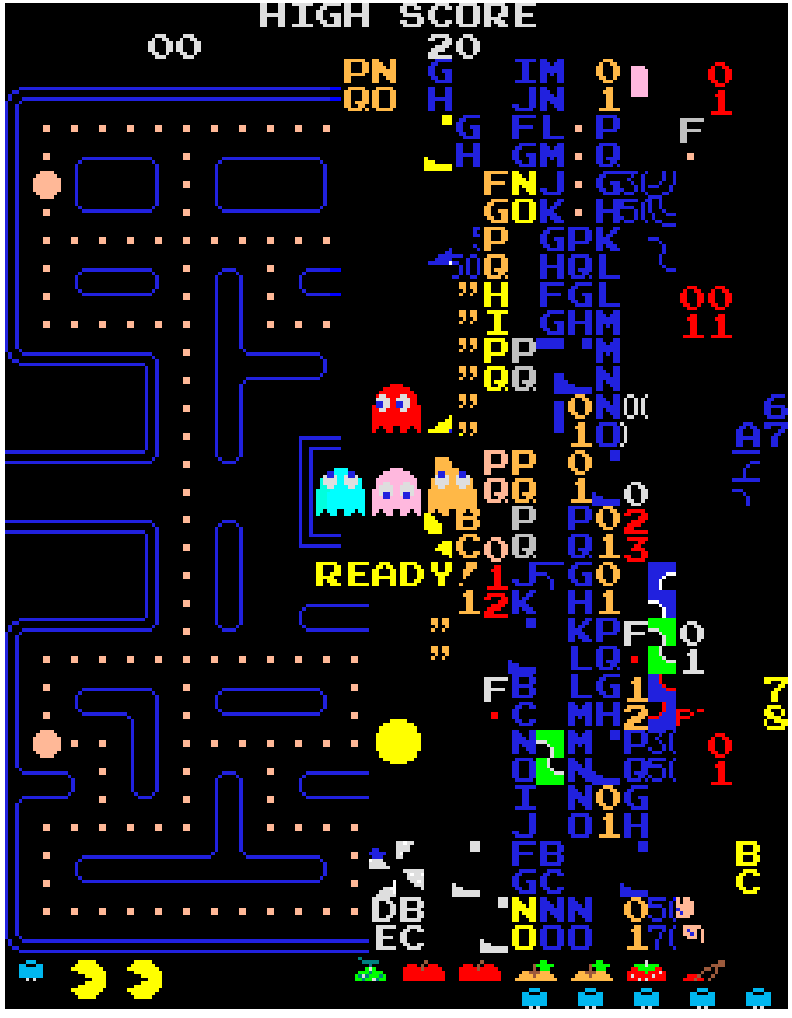

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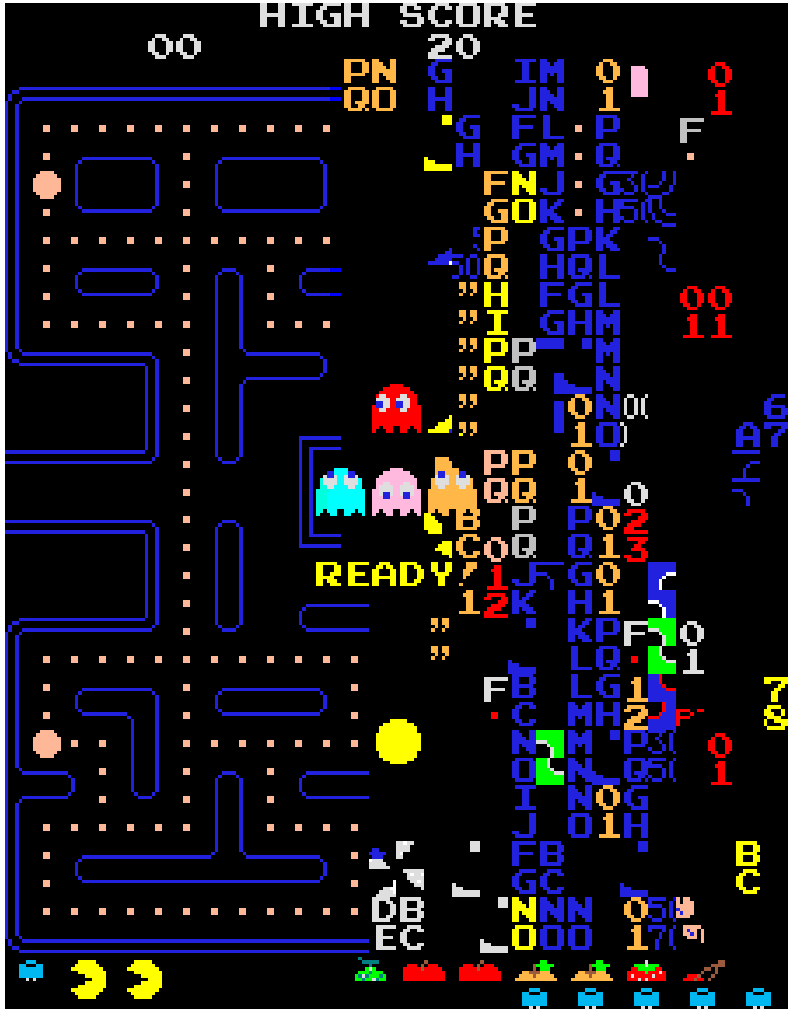

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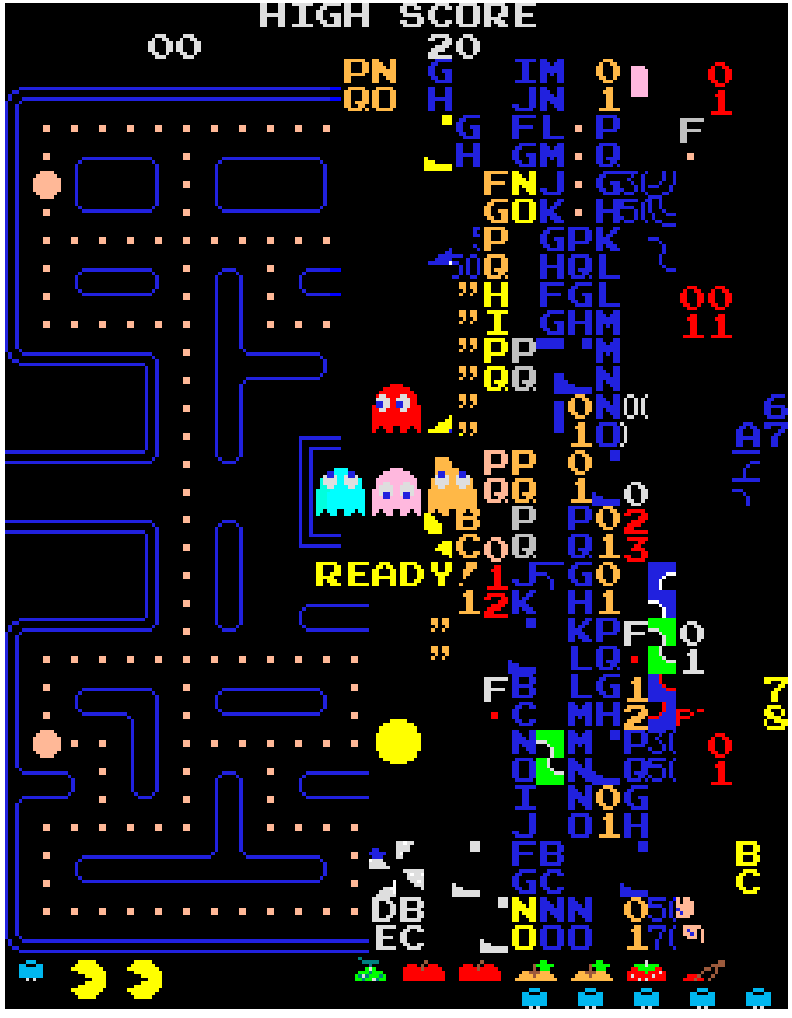

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In binary the largest 8 bit number is

$$11111111$$

which is equal to

$$2^7+2^6+2^5+2^4+2^3+2^2+2^1+2^0 = 255.$$

What to test

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There are several “parameters” in the game.

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There are several “parameters” in the game.

1. PacMan’s speed: **Slow, Medium, Fast**
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Flashing

What to test

There are several “parameters” in the game.

1. PacMan’s speed: **Slow, Medium, Fast**
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Flashing

3. Maze:

What to test

There are several “parameters” in the game.

1. PacMan’s speed: **Slow, Medium, Fast**
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Normal

Blue

Flashing

3. Maze: α, β, γ

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There are several “parameters” in the game.

1. PacMan’s speed: **Slow, Medium, Fast**
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Normal

Blue

Flashing

3. Maze: α, β, γ
4. Fruit on screen:

What to test

There are several “parameters” in the game.

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Normal

Blue

Flashing

3. Maze: α, β, γ
4. Fruit on screen:

Banana

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There are several “parameters” in the game.

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Banana

Orange

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There are several “parameters” in the game.

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Banana

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You need to test all of these parameters, so you make a chart

Your Chart

	Speed	Ghosts	Maze Type	Fruit
test 1	slow		α	

Your Chart

	Speed	Ghosts	Maze Type	Fruit
test 1	slow		α	
test 2	slow		α	

Your Chart

	Speed	Ghosts	Maze Type	Fruit
test 1	slow		α	
test 2	slow		α	
test 3	slow		α	

Your Chart

	Speed	Ghosts	Maze Type	Fruit
test 1	slow		α	
test 2	slow		α	
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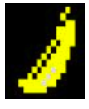
test 4	slow		β	
test 5	slow		β	
test 6	slow		β	

Your Chart

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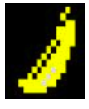
test 4	slow		β	
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$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

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There are $\underset{\substack{\uparrow \\ \textit{speed}}}{3} \times \underset{\substack{\uparrow \\ \textit{ghost}}}{3} \times \underset{\substack{\uparrow \\ \textit{maze}}}{3} \times \underset{\substack{\uparrow \\ \textit{fruit}}}{3} = 81$ different combinations!

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Playing PacMan will become really boring before you finish.

Karen's Solution to get to an Early Lunch

	Speed	Ghosts	Maze	Fruit
test 1:	slow		α	
test 2:	slow		β	
test 3:	slow		γ	
test 4:	medium		β	
test 5:	medium		γ	
test 6:	medium		α	
test 7:	fast		γ	
test 8:	fast		α	
test 9:	fast		β	

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This will test all nine possible pairs for any two parameters.

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Latin Squares

The first three columns can be written as a square:

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Fast	γ	α	β

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Each symbol occurs once in every row and column.

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This is called a **Latin square**.

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The fourth column can also be written as a square:

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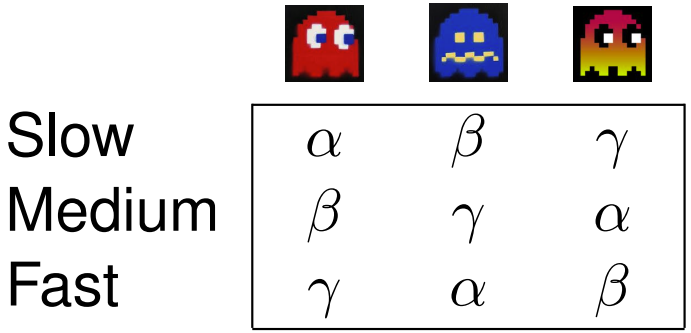

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Medium	β	γ	α
Fast	γ	α	β

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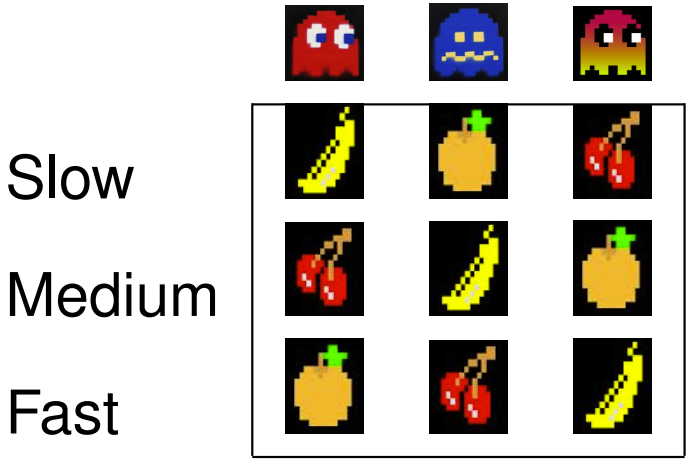
The fourth column can also be written as a square:



Slow			
Medium			
Fast			

Greco-Latin Squares

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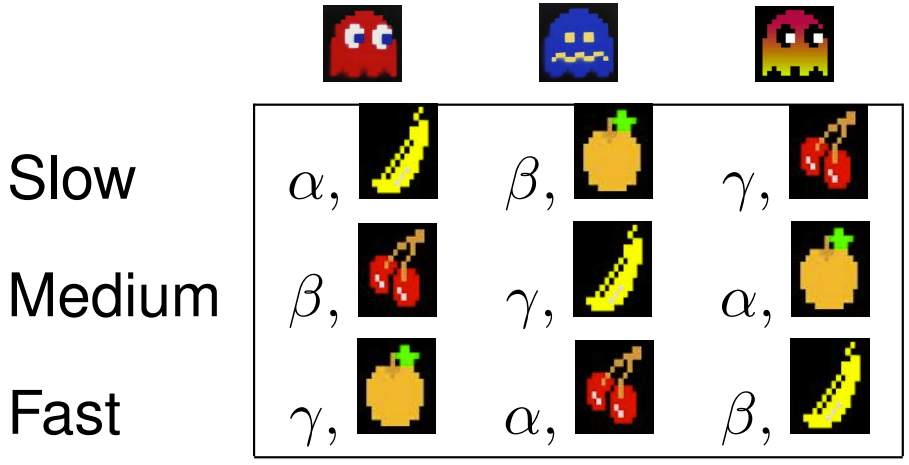

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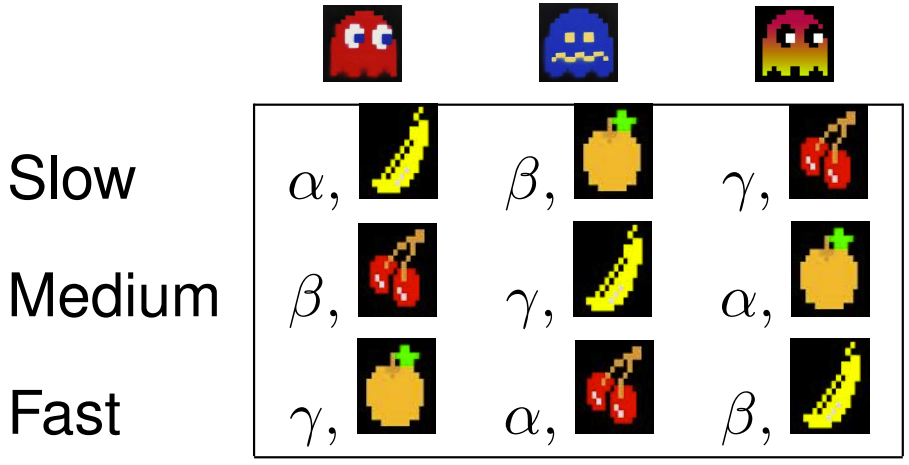

I get every maze/fruit combintation exactly once!

Everyone loves Latin Squares!

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

Sudoko games are 9×9 Latin squares.

History

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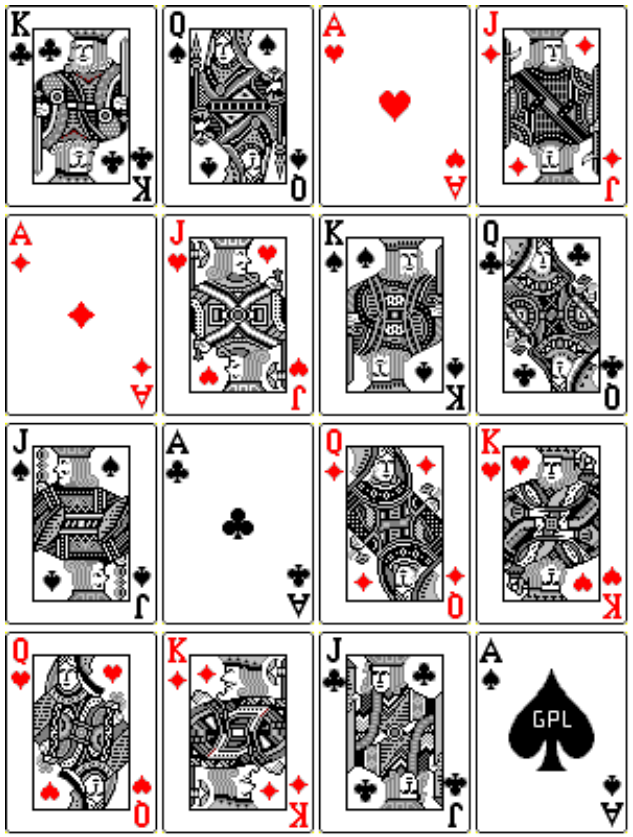

Latin Squares are good for “art”

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This is a 7×7 Greco-Latin square

Latin Squares are good for “art”

This is a 7×7 Greco-Latin square (it is a baby blanket I made for my Ph.D. supervisor’s baby).

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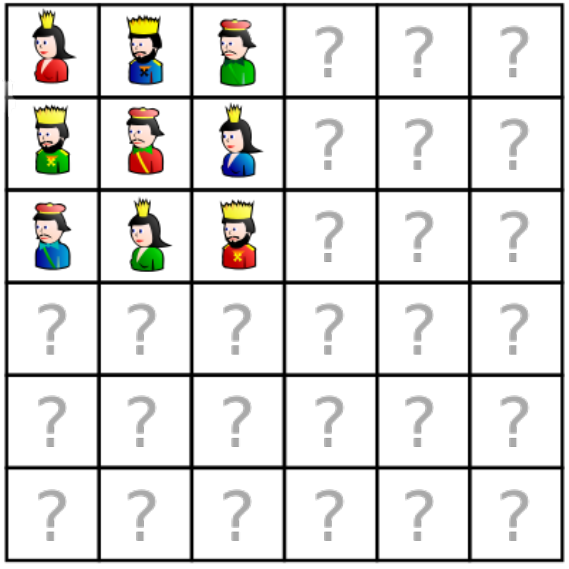

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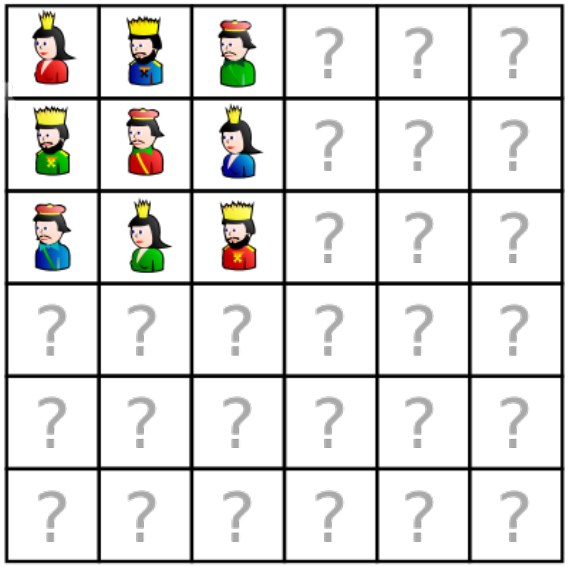

This problem is asking for a 6×6 Latin square

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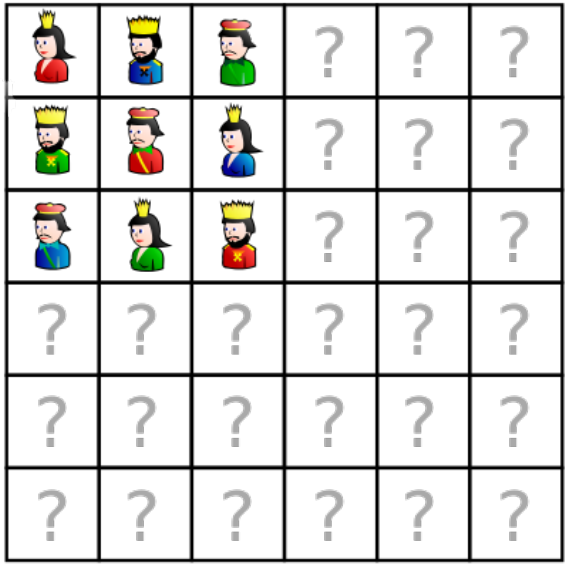

This problem is asking for a 6×6 Latin square

In 1901 it was shown that it is not possible to make a 6×6 Greco-Latin square!

10×10 Latin Square

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For a long time it was unknown if it was possible to build a 10×10 Greco-Latin square.

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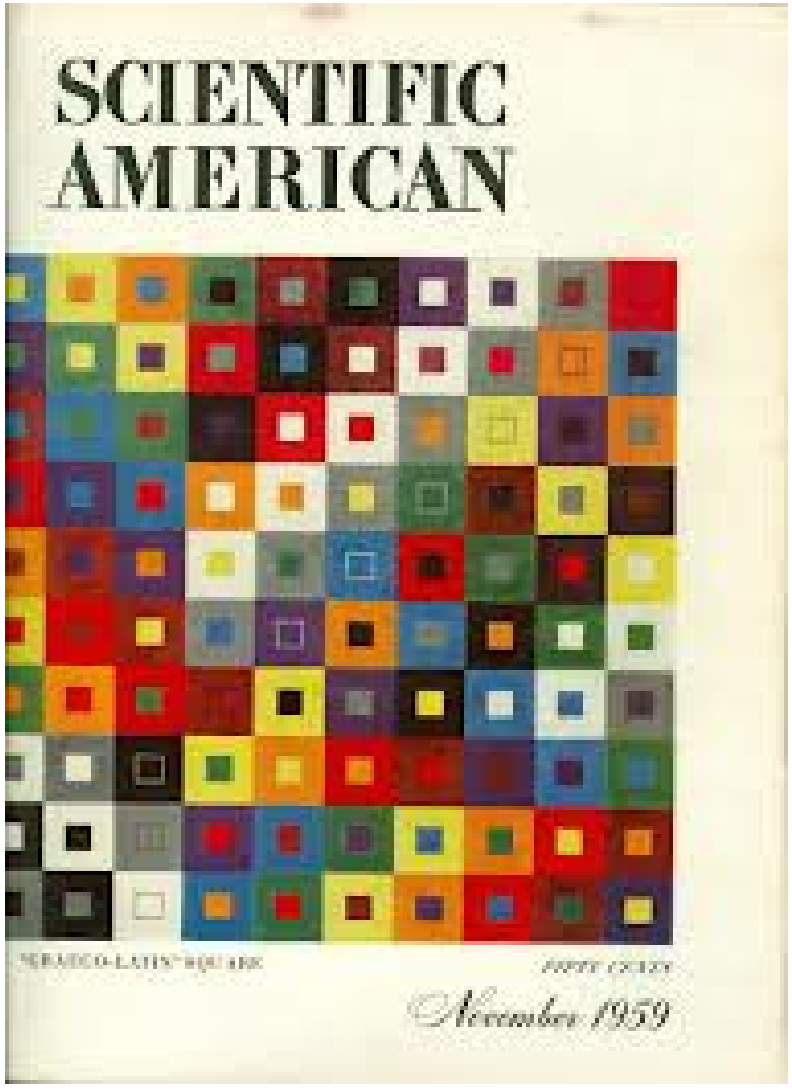

A computer in 1959?

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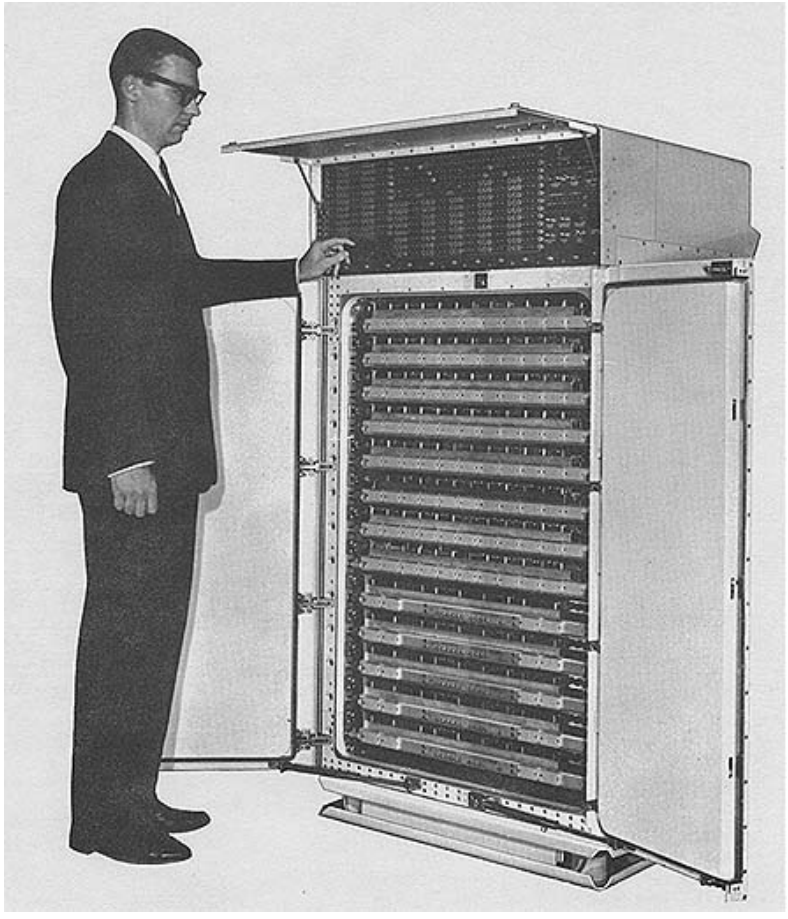

This is the type of computer that was used to find the first 10×10 Greco-Latin square.

Open Problems with Latin Squares

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We look for ways to build them:

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I can build an $n \times n$ Latin square for any n:

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2	3	4	...	n	1

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2	3	4	...	n	1
3	4	5	...	1	2

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1	2	3	$\ldots$	$n-1$	n
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$\vdots$					
n	1	2	$\ldots$	$n-2$	$n-1$

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I can also build an $n \times n$ Greco-Latin square if n is the power of a prime number.

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I can also build an $n \times n$ Greco-Latin square if n is the power of a prime number. It is easy, if you have a finite field!

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We try to count Latin squares:

Open Problems with Latin Squares

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There are two or order 2:

1	2
2	1

2	1
1	2

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Order	Number
3	12
4	576
5	161280
6	812 851 200

Open Problems with Latin Squares

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We classify Latin squares:

Open Problems with Latin Squares

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Open Problems with Latin Squares

We classify Latin squares:

1	2	3	4
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no transversal

1	4	2	3
3	2	4	1
4	1	3	2
2	3	1	4

transversal

Last Slide

Still **LOTS** of open problems with Latin squares and Graeco-Latin squares and experimental design.

Last Slide

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- Build better test sets:
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- Latin Squares are important in other aspects of math:
  - finite fields—these are important for cyptography;
  - decompositions of graphs;
  - extremal combinatorics.