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On the rate of complete convergence for weighted sums of arrays of Banach space valued random elements with application to moving average processes

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Abstract

We obtain complete convergence results for arrays of rowwise independent Banach space valued random elements. In the main result no assumptions are made concerning the geometry of the underlying Banach space. As corollaries we obtain a result on complete convergence in stable type p Banach spaces and on the complete convergence of moving average processes. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The concept of complete convergence was introduced by Hsu and Robbins (1947) as follows. A sequence of random variables $\{U_n, n \geqslant 1\}$ is said to *converge completely* to a constant c if $\sum_{n=1}^{\infty} P\{|U_n - c| > \varepsilon\} < \infty$ for all $\varepsilon > 0$. By the Borel–Cantelli lemma, this implies $U_n \to c$ almost surely (a.s.) and the converse implication is true if the $\{U_n, n \geqslant 1\}$ are independent. Hsu and Robbins (1947) proved that the sequence of arithmetic means of independent and identically distributed random variables converges completely to the expected value if the variance of the summands is finite.

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This result has been generalized and extended in several directions (see Pruitt, 1966; Rohatgi, 1971; Hu et al., 1989; Gut, 1992; Wang et al., 1993; Kuczmaszewska and Szynal, 1994; Sung, 1997; Hu et al., 1989; and Hu et al., 2001 among others). Some of these articles concern a Banach space setting. A sequence of Banach space valued random elements is said to *converge completely* to the 0 element of the Banach space if the corresponding sequence of norms converges completely to 0.

Hu et al. (1999) presented a general result establishing complete convergence for the row sums of an array of rowwise independent but not necessarily identically distributed Banach space valued random elements. Their result also specified the corresponding rate of convergence. The Hu et al. (1999) result unifies and extends previously obtained results in the literature in that many of them (for example, results of Hsu and Robbins, 1947; Hu et al., 2001; Hu et al., 1989; Gut, 1992; Kuczmaszewska and Szynal, 1994; Pruitt, 1966; Rohatgi, 1971; Sung, 1997; and Wang et al., 1993) follow from it.

Theorem 1.1 (Hu et al., 1999). *Let $\{V_{nk}, k \geqslant 1, n \geqslant 1\}$ be an array of rowwise independent random elements in a separable real Banach space and let $\{c_n, n \geqslant 1\}$ be a sequence of positive constants. Suppose that*

$$\sum_{n=1}^{\infty} c_n \sum_{k=1}^{\infty} P\{\|V_{nk}\| > \varepsilon\} < \infty \quad \text{for all } \varepsilon > 0, \tag{1}$$

$$\sum_{n=1}^{\infty} c_n \left(\sum_{k=1}^{\infty} E\|V_{nk}\|^q \right)^J < \infty \quad \text{for some } 0 < q \leqslant 2 \text{ and } J \geqslant 2, \tag{2}$$

$$S_n \equiv \sum_{k=1}^{\infty} V_{nk} \xrightarrow{\mathrm{P}} 0$$

and

$$\text{if } \liminf_{n\to\infty} c_n = 0, \quad \text{then } \sum_{k=1}^{\infty} P\{\|V_{nk}\| > \delta\} = \mathrm{o}(1) \quad \text{for some } \delta > 0. \tag{3}$$

Then

$$\sum_{n=1}^{\infty} c_n P\{\|S_n\| > \varepsilon\} < \infty \quad \text{for all } \varepsilon > 0.$$

It is implicitly assumed in Theorem 1.1 that the series S_n converges a.s.

The article Hu et al. (2001) is devoted to presenting applications of Theorem 1.1 to obtain new complete convergence results. Theorem 1.2 generalizes results of Hsu and Robbins (1947), Hu et al. (1989), Gut (1992), Kuczmaszewska and Szynal (1994), Pruitt (1966), Rohatgi (1971), Sung (1997), and Wang et al. (1993) in three directions, namely:

(i) Banach space valued random elements instead of random variables are considered.

(ii) An array rather than a sequence is considered.
(iii) The rate of convergence is obtained.

Theorem 1.2 (Hu et al., 2001). *Let $\{V_{nk}, k \geqslant 1, n \geqslant 1\}$ be an array of rowwise independent random elements taking values in a separable real Banach space $\mathscr{X}$. Suppose that $\{V_{nk}, k \geqslant 1, n \geqslant 1\}$ is stochastically dominated by a random variable X. Let $\{a_{nk}, k \geqslant 1, n \geqslant 1\}$ be an array of constants such that*

$$\sup_{k \geqslant 1} |a_{nk}| = \mathcal{O}(n^{-\gamma}) \quad \textit{for some } \gamma > 0$$

and

$$\sum_{k=1}^{\infty} |a_{nk}| = \mathcal{O}(n^{\alpha}) \quad \textit{for some } \alpha \in [0, \gamma).$$

If

$$E|X|^{1+((1+\alpha+\beta)/\gamma)} < \infty \quad \textit{for some } \beta \in (-1, \gamma - \alpha - 1]$$

and

$$S_n \equiv \sum_{k=1}^{\infty} a_{nk} V_{nk} \xrightarrow{\text{P}} 0,$$

then

$$\sum_{n=1}^{\infty} n^{\beta} P\{\|S_n\| > \varepsilon\} < \infty \quad \textit{for all } \varepsilon > 0.$$

The proof of Theorem 1.2 is rather complicated once it uses the Stieltjes integral techniques, summation by parts lemma and so on. Our initial objective was only to find a simpler proof. But it appears that we were able to establish a more general result and with simpler proof. The result presented in this paper is more general than the main result of Hu et al. (2001), since we can establish rates of convergence for moving averages which cannot be proved using Theorem 1.2.

If we assume that the Banach space has the geometric property of being of an appropriate stable type, then we can drop the condition that S_n converges in probability. For this we need the following result.

Theorem 1.3 (Adler et al., 1999). *Let $\{V_{nk}, k \geqslant 1, n \geqslant 1\}$ be an array of rowwise independent mean* 0 *random elements in a separable real stable type p $(1 < p < 2)$ Banach space. Suppose that $\{V_{nk}, k \geqslant 1, n \geqslant 1\}$ is stochastically dominated by a random element V. Let $\{a_{nk}, k \geqslant 1, n \geqslant 1\}$ be an array of constants such that*

$$\sup_{n \geqslant 1} \sum_{k=1}^{\infty} |a_{nk}|^p < \infty \quad \textit{and} \quad \sup_{k \geqslant 1} |a_{nk}| = \text{o}(1).$$

If $\lim_{t \to \infty} t^p P\{\|V\| > t\} = 0$, *then* $\sum_{k=1}^{\infty} a_{nk} V_{nk} \xrightarrow{\text{P}} 0$.

The plan of the paper is as follows. In Section 2, we recall some well known definitions pertaining to the current work. In Section 3, we apply Theorem 1.1 to obtain complete convergence for row sums with corresponding rates of convergence. As in Theorems 1.1 and 1.2, no assumptions are made concerning the geometry of the underlying Banach space. Finally, in Section 4, we present a result on complete convergence of moving averages.

2. Preliminaries

Let $(\Omega, \mathscr{F}, P)$ be a probability space and let $\mathscr{X}$ be a separable real Banach space with norm $\|\cdot\|$. A *random element* is defined to be an $\mathscr{F}$-measurable mapping of Ω into $\mathscr{X}$ equipped with the Borel σ-algebra (that is, the σ-algebra generated by the open sets determined by $\|\cdot\|$). A detailed account of basic properties of random elements in separable real Banach spaces can be found in Taylor (1978).

Let $\{V_{nk}, k \geqslant 1, n \geqslant 1\}$ be an array of rowwise independent, but not necessarily identically distributed, random elements taking values in $\mathscr{X}$. The array of random elements $\{V_{nk}, k \geqslant 1, n \geqslant 1\}$ is said to be *stochastically dominated* by a random variable X if there exists a constant $D < \infty$ such that $P\{\|V_{nk}\| > x\} \leqslant DP\{|DX| > x\}$ for all $x > 0$ and for all $n \geqslant 1$ and $k \geqslant 1$. Let $\{a_{nk}, k \geqslant 1, n \geqslant 1\}$ be an array of constants (called *weights*) and consider the sequence of *weighted sums* $S_n \equiv \sum_{k=1}^{\infty} a_{nk} V_{nk}, n \geqslant 1$.

Let $0 < p \leqslant 2$ and let $\{\theta_n, n \geqslant 1\}$ be independent and identically distributed stable random variables each with characteristic function $\phi(t) = \exp\{-|t|^p\}, -\infty < t < \infty$. The real separable Banach space $\mathscr{X}$ is said to be of *stable type p* if $\sum_{n=1}^{\infty} \theta_n v_n$ converges almost surely whenever $\{v_n, n \geqslant 1\} \subseteq \mathscr{X}$ with $\sum_{n=1}^{\infty} \|v_n\|^p < \infty$. Equivalent characterizations of a Banach space being of stable type p, properties of stable type p Banach spaces, as well as various relationships between the conditions "Rademacher type p" and "stable type p" may be found in Adler et al. (1999, Section 2).

Finally, the symbol C denotes throughout a generic constant ($0 < C < \infty$) which is not necessarily the same one in each appearance, for $x \geqslant 0$ the symbol $[x]$ denotes the greatest integer in x, and for a finite set A the symbol $\#A$ denotes the number of elements in the set A.

3. Main results

With the preliminaries accounted for, the main result may now be established.

Theorem 3.1. *Let $\{V_{nk}, k \geqslant 1, n \geqslant 1\}$ be an array of rowwise independent random elements taking values in a separable real Banach space $\mathscr{X}$. Suppose that $\{V_{nk}, k \geqslant 1, n \geqslant 1\}$ is stochastically dominated by a random variable X. Let $\{a_{nk}, k \geqslant 1, n \geqslant 1\}$ be an array of constants such that*

$$\sup_{k \geqslant 1} |a_{nk}| = \mathcal{O}(n^{-\gamma}) \quad \textit{for some } \gamma > 0 \tag{4}$$

and

$$\sum_{k=1}^{\infty} |a_{nk}| = \mathcal{O}(n^{\alpha}) \quad \textit{for some } \alpha < \gamma. \tag{5}$$

Let β be such that $\alpha+\beta \neq -1$ and fix $\delta > 0$ such that $\alpha/\gamma + 1 < \delta \leqslant 2$. If

$$E|X|^{\nu} < \infty \quad \text{where } \nu = \max\left(1 + \frac{1+\alpha+\beta}{\gamma}, \delta\right) \tag{6}$$

and

$$S_n \equiv \sum_{k=1}^{\infty} a_{nk} V_{nk} \xrightarrow{\text{P}} 0,$$

then

$$\sum_{n=1}^{\infty} n^{\beta} P\{\|S_n\| > \varepsilon\} < \infty \quad \text{for all } \varepsilon > 0.$$

Proof. Note that the result is of interest only for $\beta \geqslant -1$. Moreover, since $\delta > \alpha/\gamma + 1$, we have that $\alpha - \gamma(\delta - 1) < 0$.

First of all we verify that the series $S_n \equiv \sum_{k=1}^{\infty} a_{nk} V_{nk}$ converges a.s. Note at the outset that the stochastic domination hypothesis ensures that $E\|V_{nk}\| \leqslant CE|X|, k \geqslant 1, n \geqslant 1$ and hence for all $n \geqslant 1$, by the Beppo Levi theorem and (5)

$$E\left(\sum_{k=1}^{\infty} \|a_{nk} V_{nk}\|\right) = \sum_{k=1}^{\infty} E\|a_{nk} V_{nk}\| \leqslant CE|X| \sum_{k=1}^{\infty} |a_{nk}|$$
$$\leqslant C\, n^{\alpha} < \infty.$$

Thus, for all $n \geqslant 1, \sum_{k=1}^{\infty} \|a_{nk} V_{nk}\| < \infty$ a.s. and so for all $n \geqslant 1$ and all $K \geqslant 1$,

$$\sup_{L>K} \left\| \sum_{k=1}^{L} a_{nk} V_{nk} - \sum_{k=1}^{K} a_{nk} V_{nk} \right\| = \sup_{L>K} \left\| \sum_{k=K+1}^{L} a_{nk} V_{nk} \right\|$$
$$\leqslant \sup_{L>K} \sum_{k=K+1}^{L} \|a_{nk} V_{nk}\| = \sum_{k=K+1}^{\infty} \|a_{nk} V_{nk}\| \overset{K\to\infty}{\to} 0 \text{ a.s.}$$

Thus, for all $n \geqslant 1$, with probability $1, \{\sum_{k=1}^{K} a_{nk} V_{nk}, K \geqslant 1\}$ is a Cauchy sequence in $\mathscr{X}$ whence $\sum_{k=1}^{\infty} a_{nk} V_{nk}$ converges a.s.

Let $c_n = n^{\beta}, n \geqslant 1$. Then we only need to verify that conditions (1)–(3) (if $\beta < 0$) of Theorem 1.3 hold with $a_{nk} V_{nk}$ playing the role of V_{nk} in the formulation of that theorem. Without loss of generality, ε can be taken to be 1 and in view of (4) and (5) we can assume that

$$\sup_{k \geqslant 1} |a_{nk}| = n^{-\gamma} \tag{7}$$

and

$$\sum_{k=1}^{\infty} |a_{nk}| = n^{\alpha}. \tag{8}$$

Now to verify (1), let $b_{nj} = 1/|a_{nj}|$ (where $1/0 = \infty$),

$$I_{nj} = \{k\colon (nj)^{\gamma} \leqslant b_{nk} < (n(j+1))^{\gamma}\}$$

and

$$J_{nj} = \bigcup_{k=1}^{j} I_{nk} = \{k\colon b_{nk} < (n(j+1))^{\gamma}\}, \quad j \geqslant 1, \ n \geqslant 1.$$

Noting that by (7) $\bigcup_{j\geqslant 1} I_{nj} = \mathbf{N}$ for all $n \geqslant 1$, where $\mathbf{N}$ is the set of positive integers.

Note that for all $m \geqslant 1, n \geqslant 1$ by (8)

$$n^{\alpha} \geqslant \sum_{k\in J_{nm}} |a_{nk}| = \sum_{k\in J_{nm}} 1/|b_{nk}| \geqslant \frac{\#J_{nm}}{(n(m+1))^{\gamma}}$$

and hence

$$\#J_{nm} \leqslant n^{\alpha+\gamma}(m+1)^{\gamma}.$$

Observe that since the sets $I_{nj}, j \geqslant 1$ are disjoint, for any fixed $n \geqslant 1$, we have that

$$\sum_{j=1}^{m} \#I_{nj} = \#J_{nm} \leqslant n^{\alpha+\gamma}(m+1)^{\gamma}.$$

Therefore,

$$\begin{aligned}
\sum_{n=1}^{\infty} n^{\beta} \sum_{k=1}^{\infty} P\{\|a_{nk}V_{nk}\| \geqslant 1\} &= \sum_{n=1}^{\infty} n^{\beta} \sum_{k=1}^{\infty} P\{\|V_{nk}\| \geqslant b_{nk}\} \\
&\leqslant C \sum_{n=1}^{\infty} n^{\beta} \sum_{k=1}^{\infty} P\{|X| \geqslant b_{nk}\} \leqslant C \sum_{n=1}^{\infty} n^{\beta} \sum_{j=1}^{\infty} (\#I_{nj}) P\{|X| \geqslant (nj)^{\gamma}\} \\
&\leqslant C \sum_{n=1}^{\infty} n^{\beta} \sum_{j=1}^{\infty} (\#I_{nj}) P\{|X|^{1/\gamma} \geqslant nj\} \\
&\leqslant C \sum_{n=1}^{\infty} n^{\beta} \sum_{j=1}^{\infty} (\#I_{nj}) \sum_{k=nj}^{\infty} P\{k \leqslant |X|^{1/\gamma} < k+1\} \\
&= C \sum_{n=1}^{\infty} n^{\beta} \sum_{k=n}^{\infty} P\{k \leqslant |X|^{1/\gamma} < k+1\} \sum_{j=1}^{[k/n]} (\#I_{nj}) \\
&\leqslant C \sum_{n=1}^{\infty} n^{\beta} \sum_{k=n}^{\infty} P\{k \leqslant |X|^{1/\gamma} < k+1\} n^{\alpha+\gamma}([k/n]+1)^{\gamma} \\
&\leqslant C \sum_{n=1}^{\infty} n^{\beta} n^{\alpha+\gamma} n^{-\gamma} \sum_{k=n}^{\infty} k^{\gamma} P\{k \leqslant |X|^{1/\gamma} < k+1\}
\end{aligned}$$

$$\leqslant C\sum_{k=1}^{\infty}k^{\gamma}P\{k\leqslant|X|^{1/\gamma}<k+1\}\sum_{n=1}^{k}n^{\alpha+\beta}$$

$$\leqslant C\sum_{k=1}^{\infty}k^{\alpha+\beta+\gamma+1}P\{k\leqslant|X|^{1/\gamma}<k+1\}$$

$$\leqslant E|X|^{1+((1+\alpha+\beta)/\gamma)}<\infty \quad \text{by (6)}.$$

To verify (2), note that for any $J>(\beta+1)/(\gamma(\delta-1)-\alpha)$

$$\sum_{n=1}^{\infty}n^{\beta}\left(\sum_{k=1}^{\infty}E\|a_{nk}V_{nk}\|^{\delta}\right)^{J}=\sum_{n=1}^{\infty}n^{\beta}\left(\sum_{k=1}^{\infty}|a_{nk}|^{\delta}E\|V_{nk}\|^{\delta}\right)^{J}$$

$$\leqslant\sum_{n=1}^{\infty}n^{\beta}\left(\sup_{k\geqslant1}|a_{nk}|^{\delta-1}\sum_{k=1}^{\infty}|a_{nk}|E\|V_{nk}\|^{\delta}\right)^{J}$$

$$\leqslant C\sum_{n=1}^{\infty}n^{\beta}(n^{-\gamma(\delta-1)}n^{\alpha}E|X|^{\delta})^{J}$$

$$\text{(by (7), (8), and stochastic domination)}$$

$$=C\sum_{n=1}^{\infty}n^{\beta+J(\alpha-\gamma(\delta-1))}<\infty.$$

Finally, to verify (3) if $\beta<0$, note that for any $\lambda>0$,

$$\sum_{k=1}^{\infty}P\{\|a_{nk}V_{nk}\|>\lambda\}\leqslant\sum_{k=1}^{\infty}\lambda^{-\delta}E\|a_{nk}V_{nk}\|^{\delta}\quad\text{(by the Markov inequality)}$$

$$\leqslant C\sup_{k\geqslant1}|a_{nk}|^{\delta-1}\sum_{k=1}^{\infty}|a_{nk}|E\|V_{nk}\|^{\delta}$$

$$\leqslant Cn^{-\gamma(\delta-1)+\alpha}E|X|^{\delta}=\mathrm{o}(1)\quad\text{(by (7), (8), and stochastic domination).}\qquad\square$$

Remarks. (i) Take $\delta=1+(1+\alpha+\beta)/\gamma$ in order to obtain Theorem 1.2.

(ii) Since Theorem 3.1 is stronger than Theorem 1.2, the results of the papers Hsu and Robbins (1947), Hu et al. (1989), Gut (1992), Wang et al. (1993), Kuczmaszewska and Szynal (1994), Pruitt (1966), Rohatgi (1971), and Sung (1997) follow from it in the same way as was proved in Hu et al. (2001).

We can drop the convergence in probability condition if we assume that underling Banach space is of stable type.

Theorem 3.2. *Let $\{V_{nk}, k \geqslant 1, n \geqslant 1\}$ be an array of rowwise independent mean* 0 *random elements taking values in a separable real stable type p Banach space $\mathscr{X}$. Suppose that $\{V_{nk}, k \geqslant 1, n \geqslant 1\}$ is stochastically dominated by a random variable X. Let $\{a_{nk}, k \geqslant 1, n \geqslant 1\}$ be an array of constants such that*

$$\sup_{k \geqslant 1} |a_{nk}| = \mathcal{O}(n^{-\gamma}) \quad \textit{for some } \gamma > 0$$

and

$$\sum_{k=1}^{\infty} |a_{nk}| = \mathcal{O}(n^{\alpha}) \quad \textit{for some } 0 < \alpha < \gamma,$$

Let β be such that $\alpha + \beta \neq -1$ and fix $\delta > 0$ such that $(\alpha/\gamma) + 1 < \delta \leqslant 2$. If $p \geqslant (\alpha/\gamma) + 1$ and

$$E|X|^{v} < \infty \quad \textit{where } v = \max(1 + (1 + \alpha + \beta)/\gamma, \delta, p)$$

then

$$\sum_{n=1}^{\infty} n^{\beta} P\{\|S_n\| > \varepsilon\} < \infty \quad \textit{for all } \varepsilon > 0.$$

Proof. We need to check that

$$S_n \equiv \sum_{k=1}^{\infty} a_{nk} V_{nk} \xrightarrow{\mathrm{P}} 0.$$

To do this, we apply Theorem 1.3. Since $p \geqslant (\alpha/\gamma) + 1$, we have

$$\sup_{n \geqslant 1} \sum_{k=1}^{\infty} |a_{nk}|^{p} = \sup_{n \geqslant 1} \sum_{k=1}^{\infty} |a_{nk}|^{p-1} |a_{nk}| = \sup_{n \geqslant 1} \left(\sup_{k \geqslant 1} |a_{nk}| \right)^{p-1} \sum_{k=1}^{\infty} |a_{nk}|$$
$$\leqslant C \sup_{n \geqslant 1} n^{\alpha - \gamma(p-1)} < \infty.$$

All other assumptions of Theorem 3.1 are obviously satisfied. □

4. Complete convergence of moving average processes

In this section, we present one result about the rate of convergence of moving average processes, which follows from Theorem 3.1. The result is the generalization on the Banach space setting of the result of Li et al. (1992).

Corollary 4.1. *Assume that $\{Y_i, -\infty < i < \infty\}$ is a doubly infinite sequence of independent random elements taking values in a separable real Banach space and is stochastically dominated by a random variable X. Let $\{a_i, -\infty < i < \infty\}$ be an absolutely summable sequence of real numbers and set $V_i = \sum_{k=-\infty}^{\infty} a_{i+k} Y_k, i \geqslant 1$.*

If $1/(n^{1/\mu})\sum_{i=1}^{n} V_i \xrightarrow{P} 0$ *and* $E|X|^{(\beta+2)\mu} < \infty$, *where* $0 < \mu < 2$, $(\beta+2)\mu \neq 1$, *and* $\beta > -1$, *then*

$$\sum_{n=1}^{\infty} n^{\beta} P\left\{\left\|\sum_{i=1}^{n} V_i\right\| > \varepsilon n^{1/\mu}\right\} < \infty \quad \textit{for all} \ \ \varepsilon > 0.$$

Proof. Since all series converges absolutely, it is only a matter of renumeration that we have doubly infinite sequences and Theorem 3.1 can be applied.

Set $a_{nk} = (1/n^{1/\mu})\sum_{i=1}^{n} a_{k+i}$ and $V_{nk} = Y_k$ for all $n \geqslant 1$ and $k \geqslant 1$. Then $(1/n^{1/\mu})\sum_{i=1}^{n} V_i = \sum_{k=-\infty}^{\infty} a_{nk} V_{nk}$.

If we denote $a = \sum_{i=-\infty}^{\infty} a_i$ and $b = \sum_{i=-\infty}^{\infty} |a_i|$, then $\sup_{k\geqslant 1} |a_{nk}| \leqslant bn^{-1/\mu}$ and by the lemma of Burton and Dehling (1990) $\sum_{k=-\infty}^{\infty} |a_{nk}| \leqslant C|a|n^{1-1/\mu}$. The result follows by Theorem 3.1 with $\alpha = 1 - 1/\mu, \gamma = 1/\mu$, and $\mu < \delta \leqslant 2$. □

Furthermore, as in Theorem 3.2, the convergence in probability condition can be dropped if we assume that underling Banach space is of stable type.

Corollary 4.2. *Assume that* $\{Y_i, -\infty < i < \infty\}$ *is a doubly infinite sequence of independent mean random elements taking values in a separable real stable type* $\mu, 1 < \mu < 2$ *Banach space and is stochastically dominated by a random variable* X. *Let* $\{a_i, -\infty < i < \infty\}$ *be an absolutely summable sequence of real numbers and set* $V_i = \sum_{k=-\infty}^{\infty} a_{i+k} Y_k, i \geqslant 1$.

If $E|X|^{(\beta+2)\mu} < \infty$ *where* $\beta > -1$ *then*

$$\sum_{n=1}^{\infty} n^{\beta} P\left\{\left\|\sum_{i=1}^{n} V_i\right\| > \varepsilon n^{1/\mu}\right\} < \infty \quad \textit{for all} \ \ \varepsilon > 0.$$

Proof. The corollary follows from Theorem 3.2 and Corollary 4.1. □

Remark. (i) It is important to note that the Corollaries cannot be proved by using Theorem 1.2 for the exceedingly important cases $\mu > 1$ and $\beta \geqslant 0$. Hence, Theorem 3.1 is a more general result than Theorem 1.2.

(ii) Obviously, the case $\beta < -1$ is not of importance for Corollaries 4.1 and 4.2. However, the relatively important case $\beta = -1$ is not treated in these Corollaries. A similar result can be achieved for $\beta = -1$ with a higher order moment assumption, that is, $E|X|^{\rho} < \infty$ for some $\rho > \mu$ (all other assumptions remain in effect). In any case, we conjecture that the assumption $E|X|^{\mu} \log^{\rho}|X| < \infty$ for some $\rho > 0$ will be sufficient. □

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