

hold, where B_i is a preset symmetric positive-definite matrix and $\int_{R^N} |\nabla_{\theta_i}^{(k)} q(x; \theta_i)| dx < \infty$ ($k = 1, 2, 3$), then the guaranteed risk for the BDR $\chi = \chi(x)$ can be decomposed as

$$r_+(\chi) = r\left(\{q(\cdot; \theta_i^0)\}; \chi\right) + \delta r + O(\varepsilon_+^2), \quad \delta r = \sum_{i=1}^L \pi_i \varepsilon_i \sqrt{\alpha_i^T B_i^{-1} \alpha_i},$$

$$\alpha_i = \sum_{j=1}^L w_{ij} \int_{R^N} \chi_j(x) \nabla_{\theta_i} q(x; \theta_i^0) dx \quad (i \in S).$$

THEOREM 3. *If the random parameter distortions*

$$P_i(\varepsilon_i) = \{p_i(\cdot): p_i(x; \theta_i) = n_N(x | \mu_i, \Sigma_i^0), \quad \mu_i = \mu_i^0 + \varepsilon_i \alpha_i\}$$

hold, where α_i is a vector having the normal distribution $N_N(0, \Sigma_i^0)$, and the α_i are independent and unknown, then the risk for the Bayes classification rule equals

$$r(\chi^0) = \pi_1 \left(w_{12} + (w_{11} - w_{12}) \Phi(\Delta/2 \sqrt{1 + \varepsilon_1^2}) \right) + \pi_2 \left(w_{21} + (w_{22} - w_{21}) \Phi(\Delta/2 \sqrt{1 + \varepsilon_1^2}) \right).$$

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Volodin A. I. (Kazan'), Complete convergence in the law of large numbers for a subsequence and for a random number of random elements.

We give generalizations on weighted sums and Banach spaces of results of Gut [1] and Adler [2].

For a Banach space E , let $p(E) = \sup\{p: E \text{ is of stable type } p\}$. It is known that the interval of stable types is open, that is, if E is of type $p < 2$, then $p(E) > p$.

We shall say that a sequence of random elements (Y_n) converges completely (to zero) if for all $\varepsilon > 0$ the series $\sum_{n=1}^{\infty} P\{\|Y_n\| > \varepsilon\}$ converges.

In what follows a Banach space E is of stable type p , $1 \leq p \leq 2$; (n_k) is a sequence of strictly increasing integers; (X_k) is a sequence of independent mean-zero random elements taking values in E . Consider the complete convergence of weighted sums $T_n = \sum_{k=1}^n a_k(n) X_k$, where $\{a_k(n): 1 \leq k \leq n\}$, $n \in N$ is a triangular array of constants. Introduce the sequence $\varphi(n) = 1/\max_{1 \leq k \leq n} |a_k(n)|$.

Let (X_k) be stochastically dominated by a positive random variable ξ (that is,

$$\sup_k P\{\|X_k\| \geq t\} \leq C P\{\xi \geq t\}$$

for all $t \geq 0$) with $E \xi^r < \infty$ for some $p < r < p(E)$ and $EM(\mathfrak{K}(\psi(2\xi))) < \infty$, where $M(t) = \sum_{k \leq t} n_k$, $\mathfrak{K}(t) = \text{card}\{k: n_k \leq t\}$ and $\psi(t) = \text{card}\{n: \varphi(n) \leq t\}$, $t > 0$.

THEOREM 1. *Under these conditions T_{n_k} converges completely as $k \rightarrow \infty$.*

Now let (b_k) be a sequence of positive constants with $\limsup b_k < 1$ and

$$\sum_{k=1}^{\infty} b_k \varphi^{p-r}(n_k) < \infty,$$

and let (v_k) be a sequence of integer-valued random variables with $\sum_{k=1}^{\infty} P\{|v_k/n_k| \geq b_k\} < \infty$. Note that we do not put assumptions on the independence of (v_k) and (X_k) . Moreover, let sequence φ satisfies the condition $\varphi(2n_k) \leq C\varphi(n_k)$ for all k and some $C > 0$.

THEOREM 2. *Under these conditions T_{ν_n} converges completely as $n \rightarrow \infty$.*

These investigations will be continued by the author and A. Adler (Illinois Institute of Technology).

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S. E. Vorobeichikov and V. V. Konev (Tomsk), On detection of stationarity violation in a dynamic system.

Let (x_n, y_n) , $x_n \in R^p$, $y_n \in R^q$, be a multidimensional random process described by the system of stochastic equations

$$\begin{aligned}x_{n+1} &= \mu + Ax_n + \xi_{n+1}, & n \geq 1, \\y_n &= Qx_n + \eta_n,\end{aligned}$$

where x_n is a stable AR-process (all eigenvalues of the matrix A lie within the unit disk), y_n is the observed process, ξ_n and η_n are independent sequences of independent Gaussian random vectors with zero means and respective covariance matrices B and C $C > 0$, and x_0 is a Gaussian vector independent of (ξ_n, η_n) . Assume that at some unknown time (change time) θ the set of parameters $\lambda = (\mu, A, B, Q, C)$ of the process jump from value $\lambda_0 = (\mu_0, A_0, B_0, Q_0, C_0)$ to $\lambda_1 = (\mu_1, A_1, B_1, Q_1, C_1)$ which are known and $\lambda_0 \neq \lambda_1$. It is required to construct a procedure for detecting the change point from the observations of the process y_n that would allow are to determine characteristics of false alarms and delay time.

It is suggested that to detect the change point, forecasts should be made for y_{n+1} based on the k preceding values y_n, \dots, y_{n-k+1} in accordance with each of the alternative models. This is a cyclic procedure, i.e., after the data required for deciding whether there is a change or not has been accumulated, a new observation cycle starts. The durations of cycles are random and are determined by the time of exceeding some threshold H by the sum of conditional Kullback divergences between the conditional distributions of y_{n+1} for the competing models, given the values of y_n, \dots, y_{n-k+1} . The decision that a change (disorder) has occurred is made with the help of statistics which are increments of the functional of the least-squares method on corresponding observation cycles.

Formulas are found for the mean time between false alarms T_0 and for the mean delay T_1 in detecting the change point. The asymptotic properties of the procedure are contained in the following.

THEOREM. *Let the parameters λ_0 and λ_1 of the process (x_n, y_n) guarantee that $\max(h_0, h_1) < \infty$. Then*

- 1) $\lim_{T_0 \rightarrow \infty} (T_1 / \log T_0) = \gamma$, $\gamma = -Hh_1\kappa / \log z$,
- 2) $\lim_{a \rightarrow \infty} (T_0 / Hh_0z^a) = \rho$, $\lim_{a \rightarrow \infty} (T / Hh_1 + a\kappa) = \nu$,

where the values $h_0, h_1, \gamma, \kappa, \rho, \nu$, and z ($z > 1$) are determined by λ_0 and λ_1 , and a is the procedure parameter.

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