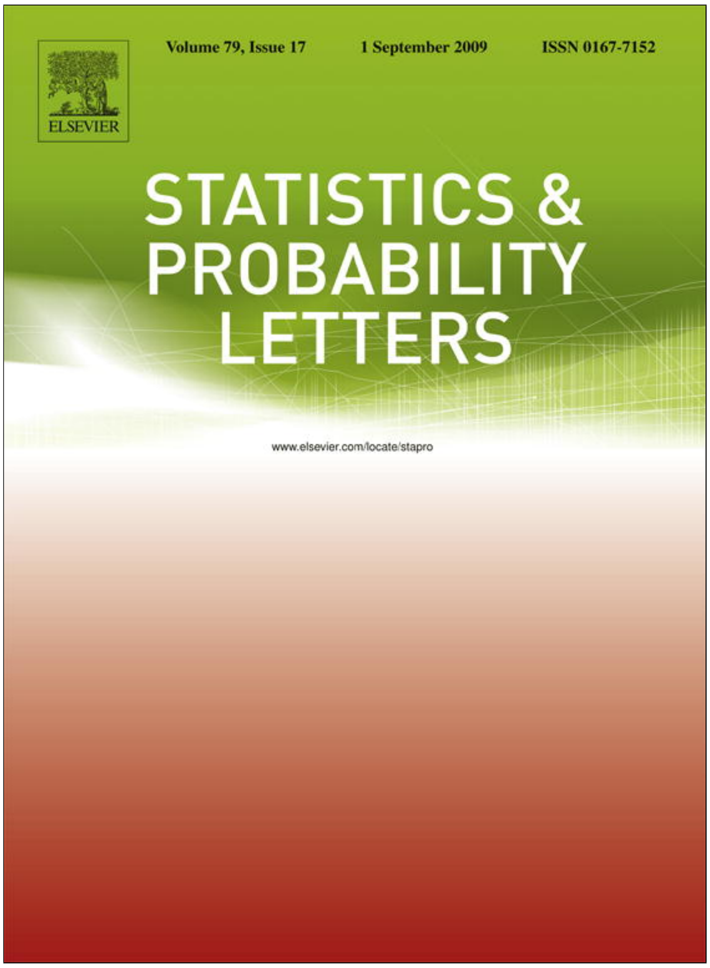
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High order approximation for the coverage probability by a confident set centered at the positive-part James–Stein estimator

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ABSTRACT

In this paper we continue our investigation connected with the new approach developed in Ahmed et al. [Ahmed, S.E., Saleh, A.K.Md.E., Volodin, A., Volodin, I., 2006. Asymptotic expansion of the coverage probability of James–Stein estimators. Theory Probab. Appl. 51 (4) 1–14] for asymptotic expansion construction of coverage probabilities, for confidence sets centered at James–Stein and positive-part James–Stein estimators. The coverage probabilities for these confidence sets depend on the noncentrality parameter τ^2, the same as the risks of these estimators. In this paper we consider only the confidence set centered at the positive-part James–Stein estimator. As is shown in the above-mentioned reference, the new approach provides a method to obtain for the given confidence set, an asymptotic expansion of the coverage probability as one formula for both cases $\tau \to 0$ and $\tau \to \infty$. We obtain the third terms of the asymptotic expansion for both mentioned cases, that is, the coefficients at τ^2 and τ^{-2}. Numerical illustrations show that the third term has only a small influence on the accuracy of the asymptotic estimation of coverage probability.

1. Introduction

The problem of confidence estimation of the mean vector $\theta = (\theta_1, \dots, \theta_p)$ for the p-dimensional normal distribution with independent components and equal variances $\sigma^2 = 1$ is considered. Let $\bar{X} = (\bar{X}_1, \dots, \bar{X}_p)$ be the vector of sample means calculated by samples of common size n from the marginal distributions. The confidence set

$$D_{\bar{X}} = \left\{\theta : n\sum_1^p (\theta_i - \bar{X}_i)^2 \le c^2\right\}$$

has the given confidence coefficient $1-\alpha$, if c^2 is the quantile of chi-square distribution with p degrees of freedom given by the relation $K_p(c^2) = 1-\alpha$, where $K_p(\cdot)$ is the chi-square distribution function.

This confidence set possesses the minimax property, but there exist other minimax sets that obtain bigger coverage probability for all values of the noncentrality parameter $\tau^2 = n\|\theta\|^2$ if $p \ge 4$. In this paper we consider one of these sets

$$D_{\delta^+} = \{\theta : n\|\theta - \delta^+(\bar{X})\|^2 \le c^2\},$$

which is centered at the positive-part James and Stein (1961) estimator given by

$$\delta^+(\bar{X}) = \left(1 - \frac{p-2}{n\|\bar{X}\|^2}\right)\bar{X}\mathbf{I}\{n\|\bar{X}\|^2 > p-2\}$$

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(it is assumed that $p > 2$). This and similar confidence sets have been investigated by many authors. (Our interest is in the asymptotic expansion of coverage probability of the true values of the vector τ.) These include Berger (1980), Hwang and Casella (1982), Casella and Hwang (1984), Hwang and Casella (1984) and some others (cf. the bibliography presented in Ahmed et al. (2006) and Efron (2006)). From the literature, the most pertinent results are those of Hwang and Casella (1982, 1984), in which they obtained approximations of the coverage probability for $\tau \to \infty$.

In Ahmed et al. (2006), a novel approach to the approximation of the coverage probability was developed, and is based on a combination of geometrical and analytical methods. It was established that $Q_p^+(\tau) = P(D_{\delta^+})$ depends on the values of the vector θ via the parameter $\tau^2 = n\|\theta\|^2$; in fact it is a decreasing function of τ^2, with $Q_p^+(\tau) = K_p(w(c,\tau)) + R_p(\tau)$, where

$$w(c,\tau) = p - 2 + \frac{c^2-\tau^2}{2} + \sqrt{\frac{(c^2-\tau^2)^2}{4} + c^2\tau^2 - (p-2)(\tau^2 - c^2)}. \tag{1}$$

The second term $R_p(\tau)$ is represented as a double integral and it is established that: $R_p(\tau) = O(\tau^2)$ for $\tau \to 0$, and $R_p(\tau) = O(\tau^{-2})$ for $\tau \to \infty$. It is important to note that the form of $R_p(\tau)$ for $\tau < c$ or $\tau > c$ is different only by an additional term in the case $\tau > c$, and this term decreases exponentially as $\tau \to \infty$.

Therefore, if we neglect the exponentially decreasing term, then we have the general formula that covers both cases of the asymptotic behaviour of τ for the coverage probability by the confidence set centered at the positive-part James–Stein estimator. This fact allows us to consider, for example, only the case $\tau \to \infty$ because the asymptotic of $Q_p^+(\tau)$ for $\tau \to 0$ will have the same form with a difference in some numerical coefficients only.

In this paper, the asymptotic behaviour of $R_p(\tau)$ is established, that is, the third order asymptotic for $Q_p^+(\tau)$ is derived. We provide the coefficients for τ^2 as $\tau \to 0$, and for $\varepsilon^2 = \tau^{-2}$ as $\tau \to \infty$. Numerical illustrations given in Ahmed et al. (2006), point out sufficiently high accuracy of the simplest approximation $Q_p^+(\tau) \approx K_p(w(c,\tau))$ for $\tau < c$ and practically acceptable accuracy in the region $\tau > c$. (The maximal error is of the magnitude 0.013 for $p = 4$ and 0.011 for $p > 4$.) The high accuracy of the approximation at zero can be explained by the fact that, contrary to the previous investigations, we did not exchange $w(c,\tau)$ by its limiting value $w(c,0)$.

Numerical illustrations for the same values of p and τ as in the paper of Ahmed et al. (2006) show that for $\tau \gg c$ and $\tau \ll 1$ the third term of the asymptotic has the magnitude of order 10^{-4}, but the general picture is such that the third term makes the approximation even worse, especially for small values of p. Therefore, the improvement of the approximation of $Q_p^+(\tau) \approx K_p(w(c,\tau))$ is still an unsolved problem.

2. Integral representations of the coverage probability

In this section we provide some results from Ahmed et al. (2006) simplifying some notations and presenting them in a convenient form for asymptotic analysis. The main idea was to represent $Q_p^+(\tau),\ \tau \geq 0$ as a probability for a region in R_+^2, which is calculated by the distribution with the density function

$$f(x,y) = \frac{1}{\sqrt{2\pi}} \frac{1}{2^{\frac{p-1}{2}}\Gamma\left(\frac{p-1}{2}\right)} (x-y^2)^{\frac{p-1}{2}-1} \mathrm{e}^{-\frac{x}{2}},$$

that takes nonzero values only for $y^2 < x$.

If $\tau < c$, then this region is bounded by the parabola $y^2 = x$ and the right branch of the hyperbola

$$y = h(x) = \frac{(x-a)^2 - c^2\tau^2 + x(\tau^2 - c^2)}{2\tau(c^2 + a - x)} = \frac{1}{2\tau}\left(a - x - \tau^2 + \frac{a(\tau^2-c^2)}{c^2+a-x}\right),$$

where $a = p - 2$ (Fig. 1). For the case $\tau > c$, to the region bounded by the parabola $y^2 = x$ and left branch of the hyperbola $y = h(x)$, the strip which is bounded by the lower branch of the parabola $y^2 = x$ and the straight line $y = (a - \tau^2 - x)/2\tau$ (Fig. 2) is added.

Point w (in the paper of Ahmed et al. (2006) $w = w_2$) on the axis OX is the null of the function h, that is, $h(w) = 0$. Let $v_{1,2}$ be the roots of the equations $h(x) = \pm\sqrt{x}$ (abscissa of intersections of corresponding branches of the hyperbola $h(\cdot)$ with the parabola $y^2 = x$), and $x_{1,2}$ be the roots of the equation $-\sqrt{x} = (a - \tau^2 - x)/2\tau$. Then for $\tau < c$ (cf. Proposition 2 in Ahmed et al. (2006)), $Q_p^+(\tau) = K_p(w) + R_p(\tau)$, where

$$R_p(\tau) = \int_w^{v_1} \mathrm{d}x \int_{h(x)}^{\sqrt{x}} f(x,y)\mathrm{d}y - \int_{v_2}^{w} \mathrm{d}x \int_{-\sqrt{x}}^{h(x)} f(x,y)\mathrm{d}y. \tag{2}$$

If $\tau > c$, then the coverage probability $Q_p^+(\tau) = K_p(w) + R_p(\tau) + J_p(\tau)$ (cf. Proposition 4 in Ahmed et al. (2006)), where

$$J_p(\tau) = \int_{x_1}^{x_2} \mathrm{d}x \int_{-\sqrt{x}}^{(a-\tau^2-x)/2\tau} f(x,y)\mathrm{d}y,$$

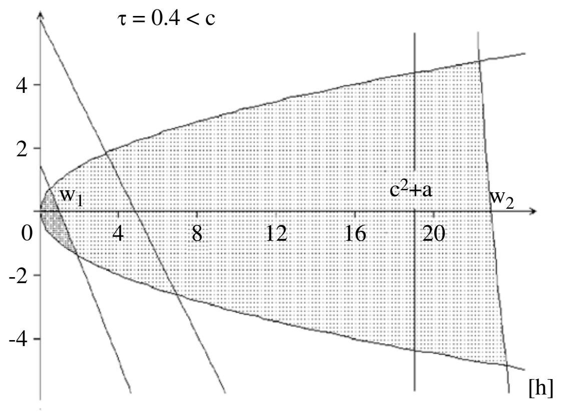

Fig. 1.

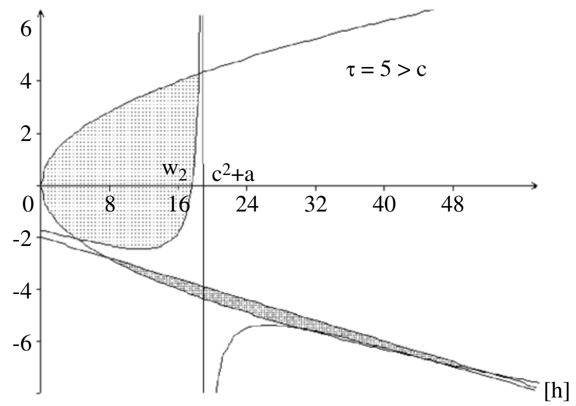

Fig. 2.

and as is shown in the proof of a theorem in Ahmed et al. (2006) for $\tau \to \infty$

$$J_p(\tau) = O\left(\tau^{p-2}\exp\{-\tau^2/2\}\right).$$

Therefore our task is to investigate the asymptotic of the difference of integrals $R_p(\tau) = J_1(v_1) - J_2(v_2)$ (cf. formula (2)). As was mentioned in Ahmed et al. (2006), functions $v_1 = v_1(\tau)$ and $v_2 = v_2(\tau)$ contract to the point w in both cases $\tau \to 0$ and $\tau \to \infty$. The direct Taylor expansion of $J_i(v_i)$ by the powers of $v_i - w$ is not possible since the derivative of this integral by v_i equals zero (recall that $h(v_{1,2}) = \pm\sqrt{v_{1,2}}$), $i = 1, 2$. Therefore, these integrals being the functions of τ, are constants as functions of the corresponding variable v_i, $i = 1, 2$. We have to use the expansion by the powers of τ for $\tau \to 0$, and by the powers of $\varepsilon = 1/\tau$ for $\tau \to \infty$. For this we need a lemma from the paper of Ahmed et al. (2006). Also note that there are some typos in the proof of a lemma in Ahmed et al. (2006). Namely, in the notation $f(x)$ we should take the reciprocal, that is instead of $f(x)$, the fraction $1/f(x)$ is defined. In the formulas for $u_{1,2}$ and $v_{1,2}$ for $\tau \to 0$, the sign $\pm$ should be exchanged by the sign $\mp$. Next, the equation for which the roots are expanded by the powers of ε, contains typos. The correct equation should be $\varepsilon^2(x-a)^2 - c^2 + x - \varepsilon^2 x c^2 = 2\varepsilon(c^2 + a - x)\sqrt{x}$.

Now we present the formulation of the result from Ahmed et al. (2006), which will be used in the derivation of the third term of the asymptotic expansion of $R_p(\tau)$.

Lemma 2.1. *If $\tau \to 0$, then*

$$v_{1,2} = w + \sum_{k=1}^{\infty} \lambda_{1,k}(w)(\mp\tau)^k, \tag{3}$$

and for $\tau \to \infty$

$$v_{1,2} = w + \sum_{k=1}^{\infty} \lambda_{2,k}(w)(\pm\varepsilon)^k, \quad \varepsilon = \tau^{-1}, \tag{4}$$

where

$$\lambda_{i,k}(w) = \frac{1}{k!}\left[\left(\frac{\mathrm{d}^{k-1} f_i^k(z)}{\mathrm{d}z^{k-1}}\right)\right]_{z=0}, \quad i = 1, 2;\ k = 1, 2, \ldots, \tag{5}$$

$$f_1(z) = \frac{(2(w-a)+z+\tau^2-c^2)}{2(c^2+a-w-z)\sqrt{w+z}}, \qquad f_2(z) = \frac{1+\varepsilon^2(2(w-a)+z-c^2)}{2(c^2+a-w-z)\sqrt{w+z}}.$$

3. Second order approximation for the coverage probability

With the preliminaries accounted for, we can formulate and prove the main result of the paper.

Theorem 3.1. *If $\tau \to 0$, then for $p \geq 4$ the following asymptotic expansion of the probability for the coverage of the true value by the confident set $D_{\delta+}$ centered at the positive-part James–Stein estimator, is true*

$$Q_p^+(\tau) = K_p(w) + \tau^2 \frac{4\lambda_{1,2} + (a-w)\lambda_{1,1}^2}{2^{(p+4)/2}\Gamma(p/2)} \cdot w^{(p-2)/2}\mathrm{e}^{-w/2} + O(\tau^3),$$

and for $p = 3$

$$Q_3^+(\tau) = K_3(w) + \tau^2 \frac{4\lambda_{1,2}w + (1-w)\lambda_{1,1}^2}{4\sqrt{2\pi w}} \cdot \mathrm{e}^{-w/2} + O(\tau^3),$$

where

$$\lambda_{1,1} = \frac{2(w-a)-c^2}{2\sqrt{w}(c^2+a-w)}, \qquad \lambda_{1,2} = \frac{w-a}{4w\sqrt{w}(c^2+a-w)^2}.$$

If $\tau \to \infty$, then for $p \geq 4$

$$Q_p^+(\tau) = K_p(w) + \varepsilon^2 \frac{4\lambda_{2,2} + (a-w)\lambda_{2,1}^2}{2^{(p+4)/2}\Gamma(p/2)} \cdot w^{(p-2)/2}\mathrm{e}^{-w/2} + O(\varepsilon^3), \tag{6}$$

and for $p = 3$

$$Q_3^+(\tau) = K_3(w) + \varepsilon^2 \frac{4\lambda_{2,2}w + (1-w)\lambda_{2,1}^2}{4\sqrt{2\pi w}} \cdot \mathrm{e}^{-w/2} + O(\varepsilon^3), \tag{7}$$

where

$$\lambda_{2,1} = \frac{1}{2\sqrt{w}(c^2+a-w)}, \qquad \lambda_{2,2} = \frac{(2w+1)(c^2+a-w)+2w}{8w\sqrt{w}(c^2+a-w)^2}.$$

Proof. We prove the asymptotic expansion for the case $\tau \to \infty$ because, as was already mentioned above, for $\tau \to 0$ the asymptotic will have the same form as for the corresponding substitution of coefficients of λ.

By formulas (3) and (4), the roots of equations $h(x) = \pm\sqrt{x}$ can be represented as $v_i = w + \Delta_i + O(\varepsilon^3)$, $i = 1, 2$, where $\Delta_{1,2} = \pm\lambda_1\varepsilon + \lambda_2\varepsilon^2 \to 0$ for $\varepsilon \to 0$. Set

$$S_1(\Delta_1) = \int_w^{w+\Delta_1} \mathrm{d}x \int_{h(x)}^{\sqrt{x}} f(x,y)\mathrm{d}y, \qquad S_2(\Delta_2) = \int_{w+\Delta_2}^{w} \mathrm{d}x \int_{-\sqrt{x}}^{h(x)} f(x,y)\mathrm{d}y.$$

By the mean value theorem for the integrals by $\mathrm{d}x$ (cf. formula (2)), it is not difficult to show that $R_p(\tau) = S_1(\Delta_1) - S_2(\Delta_2) + O(\Delta^3)$. Hence the proof reduces to a simple expansion of the functions $S_i(\Delta_i)$, $i = 1, 2$, in Taylor series by the powers of Δ keeping only two first terms.

We have $S_1(0) = 0$ and

$$S_1'(\Delta) = \int_{h(w+\Delta)}^{\sqrt{w+\Delta}} f(w+\Delta, y)\mathrm{d}y.$$

Note that $h(w+\Delta) \neq \sqrt{w+\Delta}$, since only $h(v_1) = \sqrt{v_1}$. Hence

$$S_1'(0) = \int_0^{\sqrt{w}} f(x,y)\mathrm{d}y,$$

because $h(w) = 0$.

Next,

$$S_1''(\Delta) = \frac{f(w+\Delta, \sqrt{w+\Delta})}{} + h'(w+\Delta)f(w+\Delta, h(w+\delta)) - \int_{h(w+\Delta)}^{\sqrt{w+\Delta}} \frac{\mathrm{d}}{\mathrm{d}\Delta} f(w+\Delta, y)\mathrm{d}y.$$

From this, because $f(x, \sqrt{x}) = 0$

$$S_1''(0) = -h'(w)f(w, 0) + \int_0^{\sqrt{w}} \frac{\mathrm{d}}{\mathrm{d}w} f(w, y)\mathrm{d}y.$$

Similarly we prove that

$$S_2(0) = 0, \qquad S_2'(0) = -\int_0^{\sqrt{w}} f(x, y)\mathrm{d}y,$$

$$S_2''(0) = -h'(w)f(w, 0) - \int_0^{\sqrt{w}} \frac{\mathrm{d}}{\mathrm{d}w} f(w, y)\mathrm{d}y.$$

Note that the integrals exist in the second derivatives only for $p \geq 4$. With the help of substitution $y^2 = wt$ they are reduced to integrals of beta-function, and easy calculations give us

$$\int_0^{\sqrt{w}} f(x, y)\mathrm{d}y = \frac{w^{(p-2)/2}\mathrm{e}^{-w/2}}{2^{(p+2)/2}\Gamma(p/2)}, \qquad \int_0^{\sqrt{w}} \frac{\mathrm{d}}{\mathrm{d}w} f(x, y)\mathrm{d}y = \frac{(a - w)w^{(p-4)/2}\mathrm{e}^{-w/2}}{2^{(p+4)/2}\Gamma(p/2)}.$$

These calculations allow us to write the difference of integrals in the form

$$R_p(\tau) = 2\varepsilon^2\lambda_{2,2}S_1'(0) + \frac{\varepsilon^2\lambda_{2,1}^2}{2}\left(S_1''(0) - S_2''(0)\right), \tag{8}$$

which, after easy calculations lead to formula (6).

Coefficients of $\lambda_{i,k},\ i, k = 1, 2$, are calculated by formula (5).

It is left to consider the case $p = 3$, when

$$f(x, y) = \frac{1}{2\sqrt{2\pi}} \exp\left\{-\frac{x}{2}\right\}, \quad y^2 < x.$$

We have

$$S_1(\Delta) = \frac{1}{2\sqrt{2\pi}} \int_w^{w+\Delta} \left[\sqrt{x} - h(x)\right] \exp\left\{-\frac{x}{2}\right\},$$

$$S_1(0) = 0, \qquad S_1'(0) = \frac{\sqrt{w}}{2\sqrt{2\pi}} \exp\left\{-\frac{w}{2}\right\},$$

$$S_1''(0) = \frac{1}{4\sqrt{2\pi}} \left[w^{-1/2} - 2h'(w) - w^{1/2}\right] \exp\left\{-\frac{w}{2}\right\}.$$

Similarly for the second integral we have

$$S_2(\Delta) = \frac{1}{2\sqrt{2\pi}} \int_{w+\Delta}^{w} \left[\sqrt{x} - h(x)\right] \exp\left\{-\frac{x}{2}\right\},$$

$$S_2(0) = 0, \qquad S_2'(0) = -\frac{\sqrt{w}}{2\sqrt{2\pi}} \exp\left\{-\frac{w}{2}\right\},$$

$$S_2''(0) = \frac{1}{4\sqrt{2\pi}} \left[w^{1/2} - 2h'(w) - w^{-1/2}\right] \exp\left\{-\frac{w}{2}\right\}.$$

By formula (8) we obtain (7). The theorem is proved. □

4. Concluding remarks

We can judge the influence of the third term on the asymptotic of the coverage probability according to the values presented in Table 1, in which $Q_1 = K_p(w) - 0.95$ and Q_2 are asymptotics obtained in the theorem (confidence coefficient $1 - \alpha = 0.95$). In the table we also present the error values $\Delta = Q_p^+ - K_p(w)$ obtained for the first order approximation taken from the table presented in Ahmed et al. (2006), in which the values of $Q = Q_p^+$ are given with an accuracy 10^{-4}. By the table from the current paper it is simple to find the true value of the coverage probability adding Q_1 and Δ.

Hyphens (– – –) in the columns corresponding to Q_2 mean the negative value of Q_2. Hyphens (–) in the columns for Δ (bottom part of the table) are due to the impossibility of calculating the true value of the coverage probability for very large values of τ. Underlined values of Q_1 correspond to the first value such that $\tau > c$.

According to the values in the table, it is possible to conclude that the third term of the asymptotic in the region of values of τ where it should "work" has the magnitude of the order 10^{-4} and less, and its use makes the accuracy of the

Table 1
Comparison of coverage probabilities and the accuracy of the first approximation.

τ	$p = 3$			$p = 4$			$p = 7$			$p = 10$		
	Q_1	Q_2	Δ	Q_1	Q_2	Δ	Q_1	Q_2	Δ	Q_1	Q_2	Δ
0.0	.0288	.0288	.0000	.0396	.0396	.0000	.0483	.0483	.0000	.0496	.0496	.0000
1.0	.0270	.0156	+.0008	.0383	.0285	−.0009	.0480	.0479	+.0000	.0496	.0496	+.0000
2.0	.0224	− − −	+.0020	.0345	− − −	−.0013	.0469	.0446	+.0001	.0493	.0492	+.0000
3.0	.0171	.1666	−.0038	.0289	− − −	−.0104	.0447	− − −	+.0003	.0487	.0458	+.0001
4.0	.0127	.0176	−.0084	.0230	.0234	−.0134	.0410	.0410	−.0034	.0472	− − −	+.0001
5.0	.0094	.0111	−.0070	.0181	.0182	−.0117	.0362	.0362	−.0096	.0447	.0447	−.0045
6.0	.0072	.0081	−.0055	.0142	.0142	−.0096	.0311	.0311	−.0110	.0411	.0412	−.0078
7.0	.0056	.0062	−.0044	.0113	.0113	−.0078	.0263	.0263	−.0108	.0370	.0370	−.0096
8.0	.0045	.0049	−.0036	.0091	.0091	−.0061	.0222	.0223	−.0100	.0327	.0327	−.0102
9.0	.0036	.0039	−.0030	.0075	.0075	−.0053	.0189	.0189	−.0089	.0287	.0287	−.0100
10.0	.0030	.0032	−.0024	.0062	.0062	−.0044	.0161	.0161	−.0079	.0251	.0251	−.0094
12.0	.0021	.0023	−.0017	.0045	.0045	−.0032	.0120	.0120	−.0062	.0193	.0193	−.0080
16.0	.0012	.0013	−.0010	.0026	.0026	−.0019	.0072	.0072	−.0039	.0121	.0121	−.0054
20.0	.0008	.0008	−.0006	.0017	.0017	−.0012	.0048	.0048	−.0026	.0081	.0081	−.0038
25.0	.0005	.0006	−.0004	.0011	.0011	−.0010	.0031	.0031	−.0017	.0054	.0054	−.0025
28.0	.0004	.0004	–	.0009	.0009	–	.0025	.0025	−.0014	.0043	.0043	−.0020

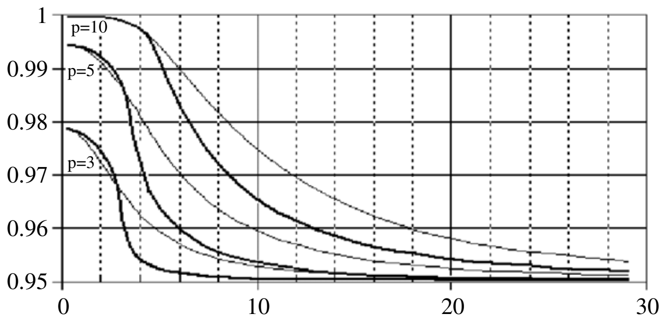

Fig. 3.

approximation worse or makes it meaningless in the neighborhood of the point $\tau = c$. Therefore an improvement of the approximation $Q_p^+(\tau) \approx K_p(w(c, \tau))$ is still an open problem.

In order to give an additional illustration of the accuracy of the approximation by Q_1, we provide a graph (cf. Fig. 3, taken from the paper of Ahmed et al. (2006)) of the functions $Q_p^+(\tau)$ (thick line) and $Q_1 = K_p(w(c, \tau))$ (thin line).

Note that in our approach to this problem, the only report the experimenter provides is the infimum; so that is what we want to approximate accurately. What would be very interesting is if we could use our approximation on a variable radius confidence set, and assume smaller balls with the same nominal coverage as the usual balls. This is still an open problem we are working on now.

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