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The Topp-Leone Generalized Exponential Power Series Distribution with Applications

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ABSTRACT In this article, a new family of distributions is introduced. It is gained by compounding a lifetime distribution with discrete distributions. It is called the Topp-Leone generalized exponential power series distribution. The distributions are utilized for reliability of parallel process with independent and identically distributed components, where the lifetime of each component shows the characteristic of the Topp-Leone generalized exponential distribution. The proposed distribution can be composed by several lifetime models. There are some special cases such as the Topp-Leone generalized exponential Poisson, Topp-Leone generalized exponential geometric, Topp-Leone generalized exponential binomial, and Topp-Leone generalized exponential logarithmic distributions. Some statistical properties of the proposed distributions are presented including the survival function, distribution function, hazard function and moments. The hazard function of the proposed distributions is categorized as decreasing, increasing, and V shaped. In this study, the maximum likelihood estimation is employed to estimate the parameter. Some real datasets are used to illustrate the goodness-of-fit depended on the generalized exponential, Topp-Leone generalized exponential, Topp-Leone generalized exponential geometric, and Topp-Leone generalized exponential Poisson distribution. The results show that its sub-model of the proposed distribution is better than selected distributions.

Keywords Maximum likelihood method; Parallel system; Power series class of distributions; Topp-Leone generalized exponential distribution

1. Introduction

The lifetime model has become well-known in many fields such as medicine, engineering, and biological organisms. Recently, various distributions have been presented for lifetime data.

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The reliability analysis is focused on the study of experimental failure components. It can often be assumed that there is a mechanism that leads to the failure of these components. It seems that there is a lack of analysis on what factors are responsible for component failure, see [2]. Consider a lifetime of a system which consists of Z components, where discrete random variable Z can have Poisson, geometric, binomial, logarithmic distribution. The general form of these selected distributions is called power series. Let Y_i be the continuous lifetime random variable of each component such as the generalized exponential or Weibull distributions. Then a random variable $X = \min(Y_1, Y_2, \cdots Y_Z)$ or $X = \max(Y_1, Y_2, \cdots Y_Z)$ represents the lifetime of components depending on the organization of components as a series or parallel system, respectively.

Some lifetime distributions including the power series distribution have been combined to lifetime model. For example, [4] presented the exponential power series (EPS) distribution which contained the exponential Poisson (EP) distribution [8], exponential logarithmic (EL) distribution [15] and exponential geometric (EG) distribution [1]. In case of the system with parallel components, the generalized exponential with family of power series distributions [9] and the complementary exponential with family of power series distribution [6] are discussed. Recently in 2016, a new attractive life time distribution is proposed in [14], namely the Topp - Leone generalized exponential (TLGE) distribution. In the application study in these article, the results suggested as the TLGE distribution is flexible and capable for modeling some phenomena in engineering field. In this work, we introduce and present the method of constructing the Topp-Leone generalized exponential power series distributions, which consists of the TLGE distribution and the power series distributions. It can be used in reliability analysis.

This work is arranged as follows. The Topp-Leone generalized exponential power series distributions are defined in Section 2. The moments and quantile function are derived in Section 3. Some special cases of the distribution are presented in Section 4. Parameters Estimation using the maximum likelihood estimation are shown in Section 5. Applications with two real datasets are given in Section 6. Section 7 concludes this research.

2. The Class of Topp-Leone Generalized Exponential Power Series Distribution

Remind that a random variable Y has the Topp-Leone generalized exponential distribution (denoted as $Y \sim \text{TLGE}(\alpha, \lambda, \beta)$), see [14], if its cumulative distribution function (cdf) is

$$G(y) = [2-(1-e^{-\lambda y})^{\beta}]^{\alpha}(1-e^{-\lambda y})^{\beta\alpha},$$

where $y > 0$, $\alpha > 0$ and $\beta > 0$ are shape parameters and $\lambda > 0$ is scale parameter. The probability density function (pdf) is

$$g(y) = 2\alpha\beta\lambda e^{-\lambda y}(1-e^{-\lambda y})^{\beta\alpha-1}[1-(1-e^{-\lambda y})^{\beta}][2-(1-e^{-\lambda y})^{\beta}]^{\alpha-1},$$

$y > 0$, $\alpha > 0$, and $\beta > 0$.

Let Z be a random variable distributed as the power series which excludes zero. Its probability mass function (pmf) is

$$P(Z = z;\, \theta) = \frac{a_z\theta^z}{C(\theta)},$$

where $z = 1, 2, \cdots, a_z \geq 0$ depends on Z, $\sum_{z=1}^{\infty} a_z\theta^z = C(\theta)$ and $\theta > 0$. Function $C(\theta)$ takes finite values and its first and second derivatives are denoted as $C'(\theta)$ and $C''(\theta)$, respectively. The inverse function of $C(\theta)$ is denoted as $C^{-1}(\theta)$. In Table 1 below (see also [10]), some special cases belong to the power series distribution (excluded zero) are presented and more details are described in [7, 12].

Table 1 Some Special Cases of the Power Series Distributions

Distributions	a_z	$C(\theta)$	$C'(\theta)$	$C^{-1}(\theta)$	Parameter space
Poisson	$z!^{-1}$	$e^{\theta}-1$	e^{θ}	$\log(\theta+1)$	$(0, \infty)$
Logarithmic	z^{-1}	$-\log(1-\theta)$	$(1-\theta)^{-1}$	$1-e^{-\theta}$	$(0, 1)$
Geometric	1	$\theta(1-\theta)^{-1}$	$(1-\theta)^{-2}$	$\theta(\theta+1)^{-1}$	$(0, 1)$
Binomial	$\binom{m}{z}$	$(\theta+1)^m-1$	$m(\theta+1)^{m-1}$	$(\theta+1)^{1/m}-1$	$(0, 1)$

Let $X_{(Z)} = \max(Y_1, Y_2, \cdots, Y_Z)$, where $Y_1, Y_2, \cdots, Y_Z$ are independent and identically distributed random variables with TLGE distribution. In this case $X_{(Z)} \mid Z = z$ has the conditional cdf as

$$G_{X_{(Z)}|Z=z}(x) = \left([2-(1-e^{-\lambda x})^{\beta}]^{\alpha}(1-e^{-\lambda x})^{\beta\alpha}\right)^{z}, \quad x > 0.$$

Thus, the Topp-Leone generalized exponential power series (TLGEPS) distribution is given by the marginal cdf of $X_{(Z)}$ is

$$F(x) = \sum_{z=1}^{\infty}\frac{a_z\theta^z}{C(\theta)}[G(x)]^z = \frac{C[\theta G(x)]}{C(\theta)} = \frac{C\left(\theta\{(1-e^{-\lambda x})^{\beta\alpha}[2-(1-e^{-\lambda x})^{\beta}]^{\alpha}\}\right)}{C(\theta)}. \tag{1}$$

Consequently, the pdf is

$$f(x) = 2\theta\alpha\beta\lambda e^{-\lambda x}[1-(1-e^{-\lambda x})^{\beta}](1-e^{-\lambda x})^{\beta\alpha-1}[2-(1-e^{-\lambda x})^{\beta}]^{\alpha-1}$$
$$\times\frac{C'\left(\theta\{(1-e^{-\lambda x})^{\beta\alpha}[2-(1-e^{-\lambda x})^{\beta}]^{\alpha}\}\right)}{C(\theta)}.$$

The survival and hazard functions are

$$s(x)=1-\frac{C[\theta G(x)]}{C(\theta)}=\frac{C(\theta)-C\left(\theta\{(1-e^{-\lambda x})^{\beta\alpha}[2-(1-e^{-\lambda x})^{\beta}]^{\alpha}\}\right)}{C(\theta)}$$

and

$$h(x)=\frac{2\theta\alpha\beta\lambda e^{-\lambda x}[1-(1-e^{-\lambda x})^{\beta}](1-e^{-\lambda x})^{\beta\alpha-1}}{C(\theta)-C[\theta G(x)]}\cdot[2-(1-e^{-\lambda x})^{\beta}]^{\alpha-1}C'[\theta G(x)].$$

In the same manner, if $X_{(1)}=\min(Y_1,Y_2,\cdots,Y_Z)$, then the cdf of $X_{(1)}$ is

$$F_{X_{(1)}}(x)=1-\frac{C\left(\theta-\theta\{(1-e^{-\lambda x})^{\beta\alpha}[2-(1-e^{-\lambda x})^{\beta}]^{\alpha}\}\right)}{C(\theta)}.$$

3. Quantiles and Moments

From (1), let $u=\dfrac{C[\theta G(x)]}{C(\theta)}$, then the quantile function of X is obtained by

$$X_u=G^{-1}\left(\frac{C^{-1}[uC(\theta)]}{\theta}\right)$$

where $u\sim\text{Uniform}(0,1)$. Note that the inverse quantile function is

$$G^{-1}(y)=-\frac{1}{\lambda}\log\left(1-(1-\sqrt{1-y^{1/\alpha}})^{1/\beta}\right).$$

A random variate X can be generated from the TLGEPS distribution using the quantile function. If $X\sim\text{TLGEPS}(\alpha,\lambda,\beta,\theta)$, then the moment generating function (mgf) of X is gathered from the cdf and pdf expansion of TLGE distribution:

$$M_X(t)=\sum_{z=1}^{\infty}\frac{a_z\theta^z}{C(\theta)}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{j}\omega_{ijk}\frac{\Gamma(\beta k+1)\Gamma\left(1-\dfrac{t}{\lambda}\right)}{\Gamma\left(\beta k-\dfrac{t}{\lambda}+1\right)} \tag{2}$$

where

$$\omega_{ijk}=\binom{z\alpha}{i}\binom{z\alpha+i}{j}\binom{j}{k}(-1)^{i+j+k}2^{z\alpha-i}.$$

By differentiating the mgf of X in (2) with respect to t and setting t equal to zero, the first moment about origin of X is

$$E(X)=\sum_{z=1}^{\infty}\frac{a_z\theta^z}{C(\theta)}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{j}\omega_{ijk}\frac{\Gamma(1)}{\lambda}[\psi(\beta k+1)-\psi(1)]$$

and the second moment about origin of X is

$$E(X^2)=\sum_{z=1}^{\infty}\frac{a_z\theta^z}{C(\theta)}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{j}\omega_{ijk}\frac{\Gamma(1)}{\lambda^2}\{\psi'(1)-\psi'(\beta k+1)+\psi(\beta k+1)-[\psi(1)]^2\}$$

where $\psi(\cdot)=\Gamma'(\cdot)/\Gamma(\cdot)$ is the digamma function and $\psi'(\cdot)$ is the trigamma function.

4. Special Cases of TLGEPS Distributions

Now we mention in some special cases of TLGEPS distributions.

4.1 The Topp-Leone Generalized Exponential Geometric Distribution

The Topp-Leone generalized exponential geometric (TLGEG) distribution is defined by using the cdf from (1) with $C(\theta)=\theta(1-\theta)^{-1}$ and $C'(\theta)=(1-\theta)^{-2}$, $0<\theta<1$, leading to

$$F(x)=\frac{(1-\theta)\nu^{\beta\alpha}(2-\nu^{\beta})^{\alpha}}{1-\theta\nu^{\beta\alpha}(2-\nu^{\beta})^{\alpha}}$$

where $\nu=1-e^{-\lambda x}$ and $x>0$. The pdf and hazard functions of the TLGEG distribution are

$$f(x)=\frac{2(1-\theta)\alpha\beta\lambda e^{-\lambda x}\nu^{\beta\alpha-1}(2-\nu^{\beta})^{\alpha-1}(1-\nu^{\beta})}{[1-\theta\nu^{\beta\alpha}(2-\nu^{\beta})^{\alpha}]^2}$$

and

$$h(x)=\frac{2(1-\theta)\alpha\beta\lambda e^{-\lambda x}\nu^{\beta\alpha-1}(2-\nu^{\beta})^{\alpha-1}(1-\nu^{\beta})}{[1-\nu^{\beta\alpha}(2-\nu^{\beta})^{\alpha}][1-\theta\nu^{\beta\alpha}(2-\nu^{\beta})^{\alpha}]}$$

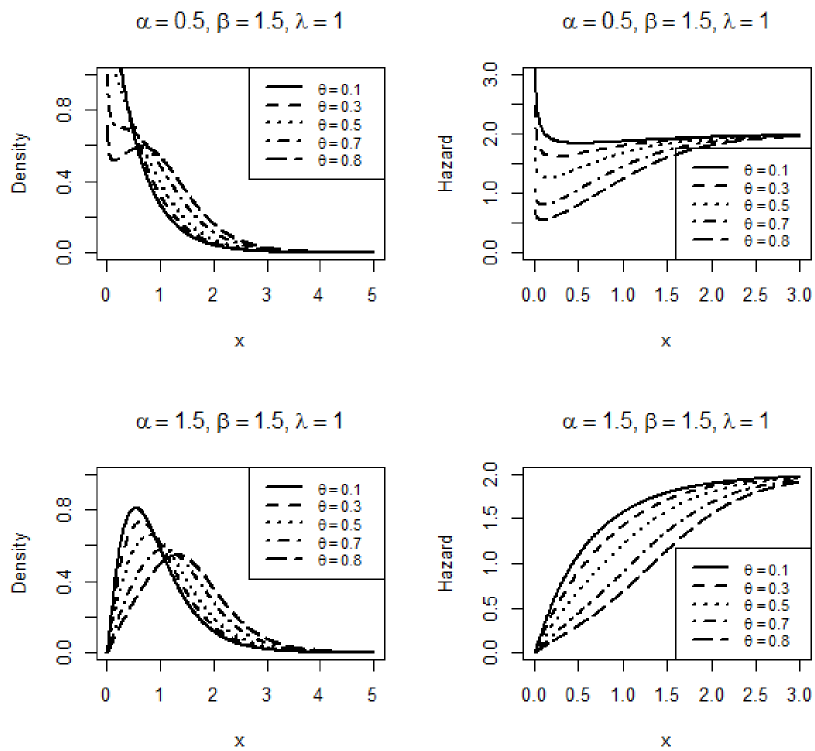

Figure 1 Plots of the TLGEG pdf and hazard function with specified parameter values

4.2 The Topp-Leone Generalized Exponential Binomial Distribution

The Topp-Leone generalized exponential binomial (TLGEB) distribution is defined by using the cdf from (1) with $C(\theta)=(\theta+1)^m-1$ and $C'(\theta)=m(\theta+1)^{m-1}$, $0<\theta<1$, leading to

$$F(x)=\frac{[\theta v^{\beta\alpha}(2-v^{\beta})^{\alpha}+1]^m-1}{(\theta+1)^m-1}$$

where $v=1-e^{-\lambda x}$ and $x>0$. The pdf and hazard functions of the TLGEB distribution are

$$f(x)=\frac{2\theta\alpha\beta\lambda m e^{-\lambda x}v^{\beta\alpha-1}(1-v^{\beta})(2-v^{\beta})^{\alpha-1}[\theta v^{\beta\alpha}(2-v^{\beta})^{\alpha}+1]^{m-1}}{(\theta+1)^m-1}$$

and

$$h(x)=\frac{2\theta\alpha\beta\lambda m e^{-\lambda x}v^{\beta\alpha-1}(1-v^{\beta})(2-v^{\beta})^{\alpha-1}[\theta v^{\beta\alpha}(2-v^{\beta})^{\alpha}+1]^{m-1}}{(\theta+1)^m-[\theta v^{\beta\alpha}(2-v^{\beta})^{\alpha}+1]^m},$$

$x>0$.

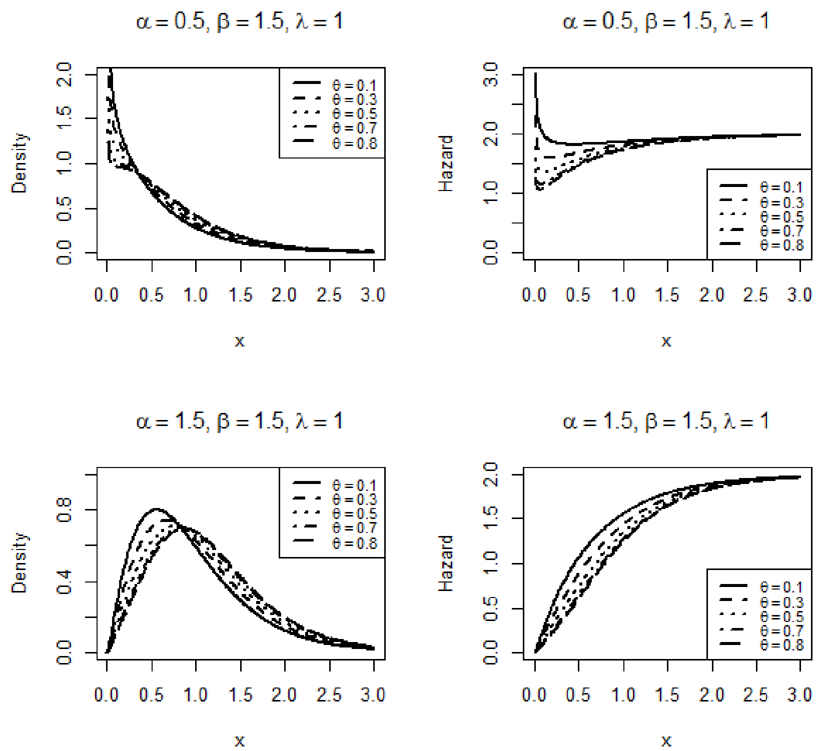

Figure 2 Plots of the TLGEB pdf and hazard function with specified parameter values

4.3 The Topp-Leone Generalized Exponential Poisson Distribution

The Topp-Leone generalized exponential Poisson (TLGEP) distribution is defined by using the cdf from (1) with $C(\theta)=e^{\theta}-1$ and $C'(\theta)=e^{\theta}$, $\theta>0$, leading to

$$F(x)=\frac{\exp\{\theta v^{\beta\alpha}(2-v^{\beta})^{\alpha}\}-1}{e^{\theta}-1}$$

where $v = 1 - e^{-\lambda x}$ and $x > 0$. The pdf and hazard functions of the TLGEP distribution are

$$f(x) = \frac{2\theta\alpha\beta\lambda e^{-\lambda x} v^{\beta\alpha - 1}(1 - v^{\beta})(2 - v^{\beta})^{\alpha - 1} \exp\{\theta v^{\beta\alpha}(2 - v^{\beta})^{\alpha}\}}{e^{\theta} - 1}$$

and

$$h(x) = \frac{2\theta\alpha\beta\lambda e^{-\lambda x} v^{\beta\alpha - 1}(1 - v^{\beta})(2 - v^{\beta})^{\alpha - 1} \exp\{\theta v^{\beta\alpha}(2 - v^{\beta})^{\alpha}\}}{e^{\theta} - \exp\{\theta v^{\beta\alpha}(2 - v^{\beta})^{\alpha}\}},$$

$x > 0$.

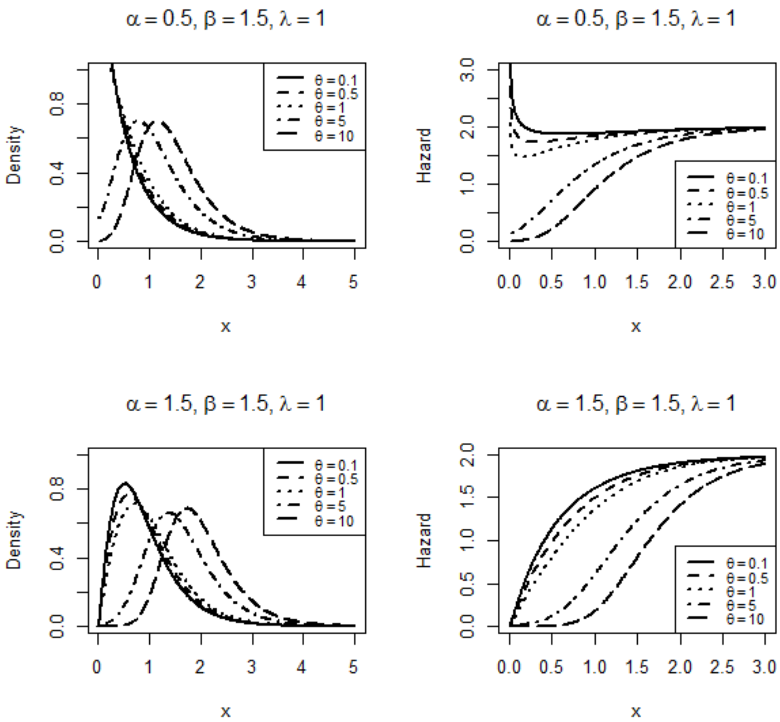

Figure 3 Plots of the TLGEP pdf and hazard function with specified parameter values

4.4 The Topp-Leone Generalized Exponential Logarithmic Distribution

The Topp-Leone generalized exponential logarithmic (TLGEL) distribution is defined by using the cdf from (1) with $C(\theta) = -\log(1 - \theta)$ and $C'(\theta) = (1 - \theta)^{-1}$, $0 < \theta < 1$, leading to

$$F(x) = \frac{\log[1 - \theta v^{\beta\alpha}(2 - v^{\beta})^{\alpha}]}{\log(1 - \theta)}$$

where $v = 1 - e^{-\lambda x}$ and $x > 0$. The pdf and hazard functions of the TLGEL distribution are

$$f(x) = \frac{2\theta\alpha\beta\lambda e^{-\lambda x} v^{\beta\alpha - 1}(1 - v^{\beta})(2 - v^{\beta})^{\alpha - 1}}{[\log(1 - \theta)][\theta v^{\beta\alpha}(2 - v^{\beta})^{\alpha} - 1]}$$

and

$$h(x) = \frac{2\theta\alpha\beta\lambda e^{-\lambda x} v^{\beta\alpha - 1}(1 - v^{\beta})(2 - v^{\beta})^{\alpha - 1}}{[\theta v^{\beta\alpha}(2 - v^{\beta})^{\alpha} - 1]\log\left((1 - \theta)/[1 - \theta v^{\beta\alpha}(2 - v^{\beta})^{\alpha}]\right)},$$

$x > 0$.

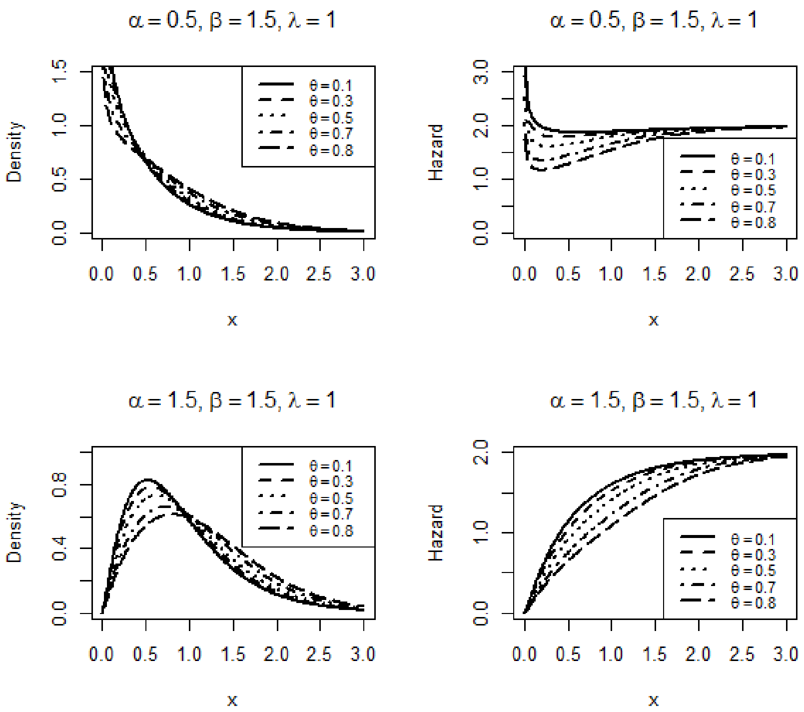

Figure 4 Plots of the TLGEL pdf and hazard function with specified parameter values

5. Parameter Estimation

In this section, parameters estimation based on the maximum likelihood estimations (MLE) will be discussed. Let $X_1, X_2, \cdots, X_n$ be independent and identically distributed TLGEPS random variables (sample) with observed values $x_1, x_2, \cdots, x_n$. Let $\Theta = (\alpha, \beta, \lambda, \theta)^T$ be a parameter vector. The log-likelihood function (ℓ) of $x_1, x_2, \cdots, x_n$ is

$$\ell = n(\log 2) + n(\log\theta) + n(\log\beta) + n(\log\alpha) + n(\log\lambda) - n\log[C(\theta)] - \lambda\sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\log(1 - v_i^{\beta})$$

$$+(\beta\alpha - 1)\sum_{i=1}^{n}\log(v_i) + (\alpha - 1)\sum_{i=1}^{n}\log(2 - v_i^{\beta}) + \sum_{i=1}^{n}\log\{C'[\theta G(x)]\}$$

where $v_i = 1 - e^{-\lambda x_i}$. Then, finding the first partial derivatives of ℓ with respect to each parameter $\alpha, \beta, \lambda,$ and θ:

$$U = \left(\frac{\partial\ell}{\partial\alpha}, \frac{\partial\ell}{\partial\beta}, \frac{\partial\ell}{\partial\lambda}, \frac{\partial\ell}{\partial\theta}\right)^T$$

where

$$\frac{\partial\ell}{\partial\alpha} = \frac{n}{\alpha} + \beta\sum_{i=1}^{n}\log(v_i) + \sum_{i=1}^{n}\log(2 - v_i^{\beta}),$$

$$\frac{\partial\ell}{\partial\beta} = \frac{n}{\beta} - \sum_{i=1}^{n}\frac{v_i^{\beta}\log(v_i)}{1 - v_i^{\beta}} + \alpha\sum_{i=1}^{n}\log(v_i) - (\alpha - 1)\sum_{i=1}^{n}\frac{v_i^{\beta}\log(v_i)}{2 - v_i^{\beta}},$$

$$\frac{\partial \ell}{\partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \frac{\beta x_i \nu_i^{\beta-1} e^{-\lambda x_i}}{1-\nu_i^{\beta}} + (\beta\alpha - 1)\sum_{i=1}^{n} \frac{x_i e^{-\lambda x_i}}{\nu_i} - (\alpha-1)\sum_{i=1}^{n} \frac{\beta x_i \nu_i^{\beta-1} e^{-\lambda x_i}}{2-\nu_i^{\beta}},$$

and

$$\frac{\partial \ell}{\partial \theta} = \frac{n}{\theta} + \sum_{i=1}^{n} \frac{C''[\theta G(x)]G'(x)}{C'[\theta G(x)]} - n\left(\frac{C'(\theta)}{C(\theta)}\right).$$

The MLEs of Θ are implemented by solving the nonlinear equations and setting $U = \vec{0}$, where $\vec{0}$ is a zero vector. In this case, there is no explicit solution of above system of nonlinear equations. Hence, the maximum likelihood estimators can be calculated by a numerical method using *optim* function in R language [13].

6. Applications

In this section, we analyze two real datasets with the TLGEG, TLGEP, TLGE and GE distributions. The first dataset is given by [3], provides the life length of maximum stresses per cycle 31,000 measured in pound force per square inch for aluminum coupons which is cut into the direction of compress at 18 cycles per second (101 samples). The second dataset from [11] presents the breaking strength data measured in GPa which consists of 100 samples which are also provided in *Adequacy Model* package [5] of R language. The proposed distribution is applied to two real datasets. We have fitted the TLGEG, TLGEP, TLGE and GE distributions. The results for both datasets are reported in Tables 2 and 3.

Table 2 Parameter Estimates, KS and AD Test Results, AIC and BIC Values for Maximum Stress Data

Parameter estimates	Distributions			
	TLGEG	TLGEP	TLGE	GE
$\hat{\beta}$	13.653	23.387	32.873	281.135
$\hat{\lambda}$	0.041	0.033	0.030	0.046
$\hat{\alpha}$	6.236	2.258	2.696	–
$\hat{\theta}$	0.978	4.803	–	–
–log LL	455.277	455.953	458.865	462.612
KS	0.049	0.077	0.096	0.108
(p-value)	(0.9715)	(0.5805)	(0.3129)	(0.1875)
AD	0.247	0.419	0.819	1.392
(p-value)	(0.9719)	(0.8298)	(0.4647)	(0.2047)
AIC	918.554	919.905	923.729	929.223
BIC	929.014	930.366	931.574	934.453

Theoretically, the most popular goodness-of-fit measures are the Kolmogorov-Smirnov (KS) test as well as Anderson-Darling (AD) test statistics which are based on empirical distribution function. For model selection, it uses minus log-likelihood (–log LL) to calculate

Akaike information and Bayesian information criteria (AIC and BIC). In Table 2 we present the fitting results of the data based on TLGEG, compared to the TLGEP, TLGE and GE distributions. The result shows that the TLGEG distribution gives the largest p-value of KS and AD test statistics. Moreover, its AIC and BIC values are the smallest. The fitted distributions for the second dataset are presented in Table 3. It shows that the TLGEG distribution also gives the smallest AIC and BIC values. In addition, the proposed distribution provides the highest p-value based on KS and AD test statistics. Therefore, the TLGEG distribution provides the better fit among selected distributions for both datasets. The plots of fitted TLGEG, TLGEP, TLGE and GE distributions with real datasets are shown in Figures 5 and 6.

Table 3 Parameter Estimates, KS and AD Test Results, AIC and BIC Values for Breaking Stress of Carbon Fibres Data

Parameter estimates	Distributions			
	TLGEG	TLGEP	TLGE	GE
$\hat{\beta}$	0.231	8.158	10.144	4.484
$\hat{\lambda}$	0.778	0.739	0.645	0.819
$\hat{\alpha}$	2.476	0.156	0.390	–
$\hat{\theta}$	0.997	5.622	–	–
–log LL	144.592	146.374	155.323	160.762
KS	0.065	0.084	0.115	0.124
(p-value)	(0.7995)	(0.4869)	(0.1392)	(0.0943)
AD	0.401	0.653	1.766	2.616
(p-value)	(0.8479)	(0.5985)	(0.1241)	(0.0432)
AIC	297.183	300.748	316.646	325.525
BIC	307.604	311.169	324.462	330.735

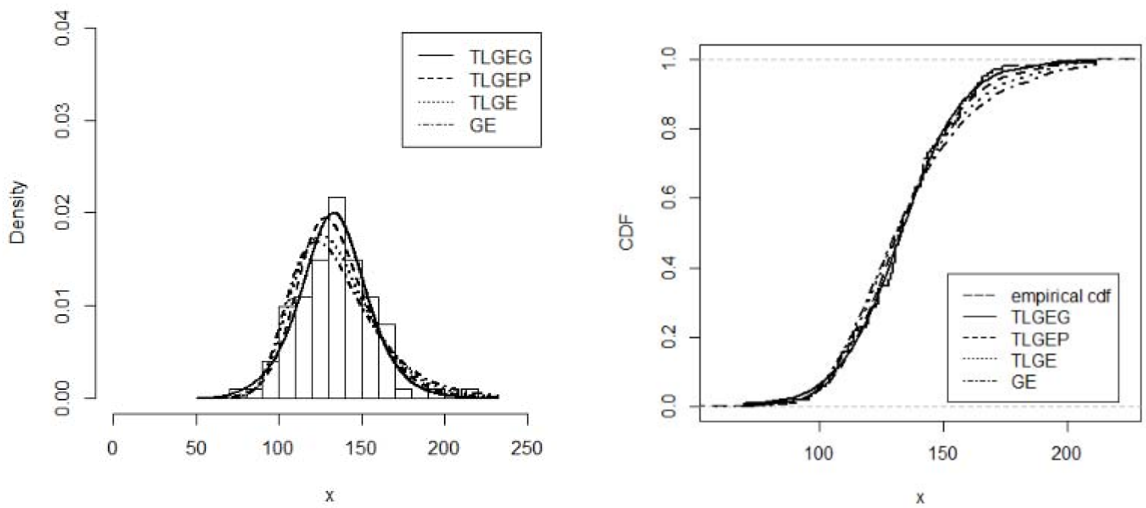

Figure 5 Plots of fitted TLGEG, TLGEP, TLGE and GE for Maximum Stress Data

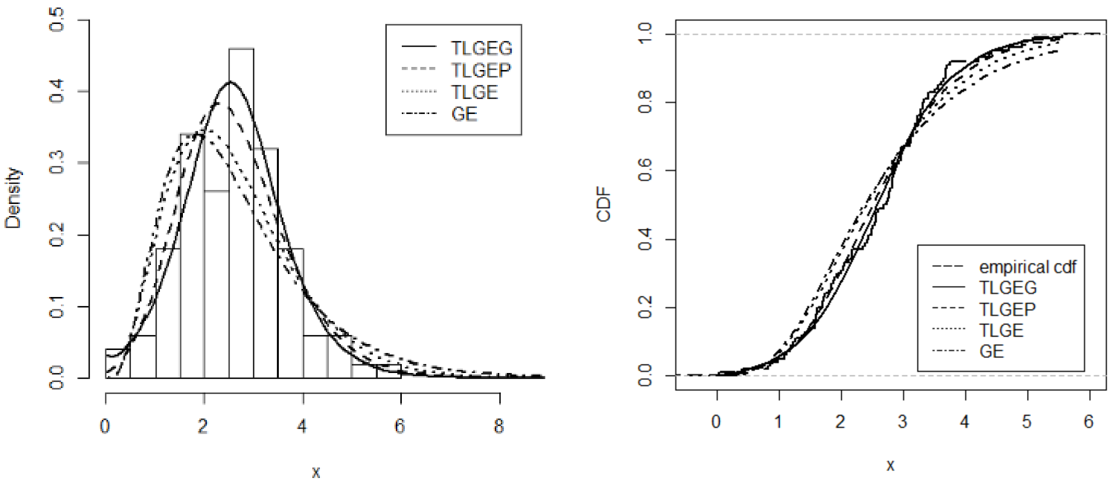

Figure 6 Plots of fitted TLGEG, TLGEP, TLGE and GE for Breaking Stress of Carbon Fibres Data

7. Conclusion

In this work, a new family of lifetime distributions is proposed, called, the TLGEPS distribution. The TLGEG, TLGEB, TLGEP and TLGEL distributions are the special cases of the proposed family of distributions. The basic mathematical properties of the proposed distribution are derived including the cumulative distribution function, probability density function, hazard function, quantile function, moment generating function and moments about origin. In this work, we mainly focus on TLGEG and TLGEP distributions. The parameters of the TLGEG and TLGEP distributions are estimated by the maximum likelihood method. The distributions are applied to two real dataset. In addition, their performance is compared with the performance of TLGE and GE distributions. The results of fitting based on AIC and BIC values indicate that the TLGEG is the best fit than other distributions. Also the *p*-value based on KS and AD test statistic of TLGEG distribution shows maximum values. It can be concluded that the TLGEG distribution is the most flexible and potentially good for fitting than others.

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