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A Review on Forecasting Models and Forecast Evaluation Criteria

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ABSTRACT In the last two decades, researchers have worked on forecasting methods for analyzing data generated from real-life problems. They used various versions of autoregressive integrated moving average (ARIMA) and generalized autoregressive conditional heteroskedasticity (GARCH) models to know about the future trend of their research questions using the data. These approaches helped them to solve future issues related to government, industry, medical, agriculture and stock market, etc. In the present study, a review of hybrid forecasting models has been studied to solve future issues and their performance based on the dataset. However, this review paper studies nine forecast evaluation criteria for selecting models related to the time series data. This review may help the researchers to know the new developments in the forecasting methods for selecting models.

Keywords AIC and MSE; ARIMA; Forecasting models; GARCH.

1. Introduction

In forecasting, ARIMA and GARCH models are a widespread class of forecasting methods that work about the future trends of time series by using past information. ARIMA(p, d, q) process of the time series $\{x_t\}$ can be defined as

$$\theta_p(B)(1-B)^d x_t = \phi_q(B)w_t, \quad (1.1)$$

where θ_p and ϕ_q are polynomials of degree p and q , respectively, w_t is the white noise with mean zero and variance σ^2 , d represents the integrated order of $\{x_t\}$ and B is a backward shift operator.

When $p = d = 0$ in (1.1), it becomes moving average (MA) model; while $q = d = 0$ in (1.1) turns into autoregressive (AR) models. If only $d = 0$ in (1.1), ARIMA becomes autoregressive moving average (ARMA) model. On the other hand, GARCH(p, q) of $\{x_t\}$ is defined as

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$$x_t = w_t \sqrt{\alpha_0 + \sum_{i=1}^p \alpha_i x_{t-i}^2 + \sum_{j=1}^q \beta_j h_{t-i}}, \quad (1.2)$$

where α_i and β_j are the model parameters of orders p and q respectively. For $q=0$, (1.2) becomes the autoregressive conditional heteroscedasticity (ARCH) models of $\{x_t\}$.

2. Review of Forecasting Models

Stevenson [26] compared the performance of ARIMA methods for foreseeing the trends of rents throughout industrial branches, commercial and offices in the United Kingdom. The judgment level and the outside pattern were used for assessing the performance of each model. The best fitting model and reliable forecasting detected through the rank of the models. Outcomes showed that forecasting performance and estimation have an optimistic relationship for the majority of cases.

Karakozova [16] used an error correlation model (ECM), an ARIMA with exogenous explanatory variables (ARIMAX) and regression models for the study of office returning of small scale market data over the period 1971-2001 in the capital of Finland (Helsinki). Results showed that the forecasting performance of ARIMAX was preferable as compared to ECM.

Earnest *et al.* [10] worked on the data of the SARS outbreak in the provincial hospital in the republic of Singapore using ARIMA model. A maximum likelihood estimation was used to study the historical dataset using the Kalman filter. Results showed that ARIMA(1,0,3) is appropriate for accurate forecasting of severe acute respiratory syndrome (SARS) in terms of mean absolute percentage error.

Pantelidis and Pittis [24] presented the low foresee performance of the GARCH(1, 1) model as compared to a naive model (homoscedastic) in several experimental studies. MSE was used to check the forecasting reliability of the models that was based on true conditional volatility and squared shocks. Apart from it, Monte Carlo Simulation experiments was conducted to evaluate the size and power properties of statistical procedures for choosing the GARCH(1, 1) model over the miss-specified naive model as homoscedastic is the one that experiences the problem of size distortion during the performance of nested models. Finally, the forecasting accuracy of the GARCH(1, 1) model is 60% as compared to 40% of the naive models on five bilateral exchange rates for US dollars.

Ediger and Akar [11] evaluated the forthcoming trend in the initial energy requirement of Turkey via ARIMA and seasonal ARIMA models for the period 2005-2020. Ten energy sources out of 12 have a downward trend in the yearly average growth rate of energy in all situations while animal plants and wood remain the same. Results also showed that fossil energy resources will constantly contribute to leading role in the future energy needs of Turkey.

Chen *et al.* [6] presented a one-week prediction of offenses against the property in a city of China. Fifty weeks of property offenses data were used to compare the ARIMA model with the holt two-parameter exponential smoothing and the simple exponential smoothing. It showed that the ARIMA model is more convenient than exponential smoothing in terms of forecasting clarity and

adjustment. Error measurements of the models were checked via the root mean squared error (RMSE) and mean absolute percent error (MAPE).

Fahimifard *et al.* [12] compared linear and non-linear forecasting models for predicting future trends in the exchange rate of Iran Rail/EURO and Rail/\$US. Authors selected Artificial Neural Network (ANN) and Adaptive Neuro-fuzzy inference system (ANFIS) as nonlinear models while ARIMA and GARCH are linear models. The accuracy of these models was assessed via mean absolute deviations (MAD), R-Squared and RMSE. Results showed that ANFIS dominated as compared to other approaches on the historical dataset.

Mariella and Tarantino [22] spatial-temporal conditional autoregressive (STCAR) model for the trust of spatial between the sites and temporary addiction to realizations during the estimation of each spatial position in the time domain. The introduction of the hypothesis of the hierarchical model in the STCAR model enhances the quality of results.

Amado and Terasvirta [3] applied a conditional correlation to the GARCH model with a non-stationary component that was related to the S&P composite index and traded. Lagrange multiplier was used to find the parametric design of non-stationary elements in the modeling approach. Results indicated that the multivariate conditional correlation GARCH approach is useful for the data as compared to DCC-GARCH, STCC-GARCH, TVCC- GARCH and the others techniques because a number of deterministic modifications in the unconditional variances improve the quality of multivariate conditional correlation GARCH approach.

Ibrahim *et al.* [15] studied autoregressive fractional integrated moving average strategy for the mean daily temperature series of the Sokoto metropolis. The performance of the ARFIMA (3, 0.62, 1) and ARFIMA (1, 0.62, 3) was beneficial for outlining, make it clear and forecasting the temperature. The authors also highlighted the importance of forecasting temperature for farming and atmospheric temperature etc.

Liu *et al.* [20] proposed a forecasting method for adapting an analysis slide window of fuzzy time series model (WTVS) to train the trend predictor in the training phase, and uses these trend predictors to generate forecasting values in the forecasting phase. Data preprocessing, trend training and the load forecasting are the key building blocks of the WTVS model. In the preprocessing stage the founder's effort to reduce the effect of random factors via leveling the dataset. Cyclic factors and the previous weighted data are used in the remaining two components. Outcomes indicate that the performance of WTVS models is better than the TVS models.

Corliss [8] focused on the elementary theories of the ARIMA model and the advancement of the non-temporal ARIMA in statistical analysis. They also mentioned that non-temporal ARIMA process used for research in biosciences, behavior sciences, business and other applications.

Vishwakarma [28] considered the ARIMA, ARIMAX including generalized autoregressive conditional heteroskedastic (ARIMAX-GARCH) to know about forecasting trends and changing points in Canadian real estate sector for the period 2002-2011. Outcomes showed that ARIMAX models is superior to the other two. Overall, three models cannot use for long term forecasting in an unstable environment.

Arumugam and Anithakumari [5] applied FARIMA model on the data of natural rubber

production for forecasting demand for the production. AIC was used to check the best-fitted kind of FARIMA for the dataset and it was concluded that FARIMA (2,1,2)(1,1,1)₁₂ perform better than other kinds of FARIMA forms.

Ahmed *et al.* [2] developed James-stein shrinkage and pretest methods for estimating the regression coefficients of the spatial conditional autoregressive model under the changeable facts that are related to regression coefficients. They applied these methods to Boston housing price and applied the bootstrapping technique to calculate relative MSE and prediction error. Numerical results showed that the pretest and James-stein shrinkage estimators protect in opposition to the hazard conditions of regression coefficients.

Zhang *et al.* [30] applied the hybrid approach for multi-stage ahead flow of traffic prediction for the highway network along with real life dataset that was based on the concept of statistical imbalance and spectral analysis. The data was split into three parts, an intraday by establishing a spectral investigation approach, an imperative section through ARIMA and a changeability evaluation with the GJR-GARCH model. Experimental outcomes showed that the planned hybrid technique is better than ARIMA and GARCH in terms of MAPE, RMSE, PICP and MPIL for forecasting the accuracy of the dataset.

Pahlavani and Roshan [23] worked on ARIMA and ARIMA-GRACH methods to investigate the exchange rate of Iran against U.S Dollar. The central bank of Iran provided the data during the period of March 2014 to June 2015. Initially, they experienced the problem of non-stationary exchange rate series that become stationary series by transform to exchange rate return. They used RMSE, MAE and TIC Criteria to determine the best-fitted model for this data. Results showed that ARIMA ((7,2),(12))-EGARCH(2,1) give the smallest value of RMSE, MAE and TIC as compared to ARIMA-GARCH, ARIMA-IGARCH and ARIMA-GJR models.

Hossian [14] proposed shrinkage estimators approach with generalized autoregressive conditional heteroscedastic error for linear regression model under the limitation of regression parameters lie on a subspace. Large sample theory was applied to develop the theoretical biases and risks of these estimators. Monte Carlo simulation experiments showed that the shrinkage estimators perform better than the full model estimator if the shrinkage dimension is greater than two.

Correa *et al.* [9] worked to increase the forecast performance and accuracy of forecasting model. They developed wavelet ARIMA with exogenous variables and generalized autoregressive conditional heteroscedasticity models that perform better than ARIMA-GARCH and Artificial Neural Network on the time series dataset that was related with dam displacement in Brazil. One important characteristic of the new approach is that the utilization of exogenous variables improves the efficiency of the new approach over the univariate method. This development provides assistance to raise the prophetic performance.

Uwilingiyimana *et al.* [27] worked on the inflation rate of Kenya for the monthly data from 2004-2014 by using the ARIMA-GARCH model. Experimental results of the monthly data series displayed that the combined approach of ARIMA(1,1,12) and GARCH(1,2) is much better than the existing methods of forecasting in terms of optimum outcomes, more effective estimation and forecasting accuracy.

Grzesica and Wiecek [13] applied spectral analysis to seek the fluctuation patterns and prediction of the modern forecasting models. Authors considered a certain interval and trigonometric functions as the basic hypothesis of the models. Numerical outcomes indicate that the accuracy of the proposed approach is at least three times greater as compared to the brown model and ARIMA. Patowary and Barman [25] used a seasonal autoregressive integrated moving average (SARIMA) model on the quarterly data of tuberculosis detection rate in the Dibrugarh region of India for the period 2001-2011. The conclusion of the experimental results showed that SARIMA $(0,0,0) \times (1,1,0)_4$ is properly suited to the quarterly data. The lowest difference between the observed and expected detection rate showed that all basic assumptions of model adequacy are true for this data.

Wali *et al.* [29] studied space and manufacturing of cotton in India via the ARIMA model for seeing the trend in the dataset during the period 1950-51 to 2015-16. The best-fitting form of the ARIMA model was determined via equating mean absolute percentage error (MAPE), Akaike information criterion (AIC), Normalized BIC, Schwartz's Bayesian Criterion (SBC) and maximum value of R^2 . The study proved that two types of ARIMA models were appropriate for the historical dataset which was ARIMA (1, 1, 1) and ARIMA (0, 1, 0).

Lac and Hossian [19] worked on the random-effects model for longitudinal data. They established a non-penalty shrinkage estimation approach with autoregressive error when some of the regression coefficients may be under linear restrictions. Monte Carlo simulation techniques were used to examine the performance of the restricted, shrinkage, positive shrinkage and LASSO estimators with respect to unrestricted estimator.

Ardia *et al.* [4] considered Markov switching GARCH (MSGARCH) model for forecasting risk of management perspectives. The left tail forecast becomes better by using the uncertainty of parameters in single regime model. Ahasan *et al.* [1] introduced a new approach of time series model that combines wavelet GARCH(1, 1) for global climate data. The efficiency of the Wavelet-GARCH (1, 1) is measured by the SC, AIC, RMSE and Hannan-Quinn Criterion. The results showed that proposed approach perform better than GARCH (1, 2) in terms of accuracy of the model. Kiregu *et al.* [17] developed point change modeling in GARCH models and analysis of forecasting implemented on the USD/KES rates data of the central bank of Kenya for the period 2005-2018. Two kinds of change points used to check the performance of TGARCH, GJR-GARCH and PGARCH models. Outcomes indicate that the modified version of GARCH model performs better than the other.

Maqsood *et al.* [21] examined the rate of inflation in Pakistan for the period 1998-2003 ARMA, autoregressive fractionally an integrated moving average (ARFIMA) and GARCH models. The performance of three univariate techniques verified on food and non-food inflation components of the data in terms of AIC, SC, RMSE and SE criteria. It demonstrated that the performance of GARCH is superior to the other two approaches.

Kushwaha and Pindoriya [18] worked on the hybridization of Seasonal autoregressive integrated moving average and random vector functional link neural network for forecasting the data of solar PV power generation. The motivation behind the hybridization of forecasting models

was to provide the actual-time equilibrium functioning for energy consumers and distributors. Decomposition of wavelet supported for hybridization of ARIMA and RVFL models.

3. Forecast Evaluation Criteria's of Forecasting Models

3.1 Akaike's Information Criteria (AIC)

This evaluation criteria was formulated by Japanese statistician Hirotugu Akaike in 1974. Firstly, it was used in information theory. Today, AIC is applied widely by researchers for the study of time series models. The selection of the model is the most challenging task in statistical inference that may be solved via AIC. It can be defined in the following way:

$$AIC = 2k - 2\ln(\hat{L}),$$

where k is the number of estimated parameters and \hat{L} is the greatest value of the likelihood function of the model.

3.2 Bayesian Information Criterion (BIC or SIC)

Gideon E. Schwarz introduced Schwarz Information Criteria but it is not appropriate in the complex set of models but better than AIC.

$$BIC = k \ln(n) - 2\ln(\hat{L}),$$

where n is the number of observations in the model.

3.3 Normalized BIC

In terms of normal distribution BIC can be defined in the following way: $BIC = n \ln(\hat{\sigma}_e^2) + k \ln(n)$, where $\hat{\sigma}_e^2$ is called Error Variance.

3.4 Deviance Information Criteria (DIC)

It is an expansion of Akaike's information criteria and superior to BIC as it is smoothly estimated from the samples that is created via Markov chain Monte Carlo Simulation.

$$DIC = p_D + \overline{D(\theta)},$$

where

$$p_D = \frac{1}{2} \overline{\text{Var}D(\theta)} \quad \text{and} \quad D(\theta) = -2\log[p(y|\theta)] + C,$$

y is the data, θ are unknown parameters and C is constant.

3.5 Hannan Quinn Information Criterion (HQC)

It is an optional approach to AIC and BIC for the selection model in statistics.

$$HQC = -2L_{\max} + 2k \ln[\ln(n)],$$

where L_{\max} represents likelihood function.

3.6 Focused Information Criterion (FIC)

Claeskens and Hjort [7] introduced Focused Information Criterion in 2003 but it does not work to find the general fitness of the model like others (AIC, BIC, and DIC). It emphasizes the primary parameter setting of the models.

3.7 Mean Squared Error (MSE)

It is widely used to evaluate the quality of forecasting models and eliminate a number of predictor variables without losing the predictive ability of forecasting models in time series analysis. The value of MSE is nearest to zero indicates the quality of the forecasting model. The average squared difference between the predicted values and the true values is known as MSE.

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2,$$

where \hat{Y}_i is predicted value of the data. The square root of MSE is called the root mean squared error (RSME)

3.8 Mean Absolute Percentage Error (MAPE)

The accuracy of forecasting models is predicted via mean absolute percentage error in time series. It provides useful information about the trend of time series during the forecasting. It applied as a loss function for regression problem but the major drawback of this evaluation is that it cannot work with a zero value in time series.

$$\text{MAPE} = \frac{100\%}{n} \sum_{i=1}^n \left| \frac{A_i - F_i}{A_i} \right|,$$

where A_i is true value while F_i is a forecast value.

3.9 Mean Absolute Error (MAE)

It is assessing the forecasting error in time series analysis as below:

$$e(t) = y(t) - \hat{y}(t | t-1),$$

where $y(t)$ represents observation and $\hat{y}(t | t-1)$ show forecast of $y(t)$ based on all the previous observations. In statistics, the mean absolute error is defined in the following way:

$$\text{MAE} = \frac{1}{n} \sum_{i=1}^n |y_i - x_i|,$$

where y_i is predicted value and x_i is true value.

4. Future Scope of Proposed Work

Presently, the use of forecasting models are growing to solve the future issue in the field of government, medical, agriculture, industry, stock market, transportation, etc. The appropriate selection of forecasting models and forecast evaluation criteria are required to settle the upcoming

challenge. The proposed work reviews the latest progress in the field of forecasting that can contribute to help the researchers and experts to examine the latest advances in this area and to introduce another version of forecasting models.

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