ON THE AUTOREGRESSIVE NATURE OF ARCTIC SEA ICE CONCENTRATIONS

JOSEPH M. PIWOWAR
ELLSWORTH F. LEDREW

Waterloo Laboratory for Earth Observations
Department of Geography
University of Waterloo
Waterloo, ON N2L 3G1
Canada

piwowar@uwaterloo.ca

ABSTRACT

Many general circulation model experiments show amplified warming in the polar regions as one of the strongest responses to enhanced atmospheric greenhouse gas concentrations. In light of this, there has been speculation that a spatially coherent pattern of high-latitude temperature trends and associated cryospheric responses could be early indicators of climatic change. Although considerable effort is being made to examine the linkages between atmospheric anomalies and the sea ice regime, the month-to-month and year-to-year variations of Arctic sea ice remain poorly understood and insufficiently modelled.

Ice persistence and thermal inertia suggest that month-to-month sea ice concentrations are strongly correlated - e.g. present conditions are a reliable indicator of future conditions - but this has never been proven statistically. We use statistical times series analysis to characterize the variability of monthly Arctic sea ice concentrations from a nine-year satellite passive microwave record acquired between 1978 and 1987. Our focus is on documenting the autoregressive nature of the ice cover rather than identifying climatically significant changes. Interestingly, we find that only 62% of the Arctic sea ice zone is adequately represented by first-order autoregressive models, meaning that the sea ice concentrations in any given month are closely related to that of the preceding month. For the remaining 38% we identify a variety of AutoRegressive - Moving Average (ARMA) time series models that differ from our geophysical expectations.

Our findings provide a deeper understanding of the cryospheric processes at work in the Arctic. They can lead to the development of better sea ice models and be used to develop ice concentration forecasts with statistical confidence. Our results also identify those polar regions where climatically-induced changes may be most prevalent in the future.
INTRODUCTION

The earth's polar regions play a critical role in global change scenarios. Polar climates exist through a delicate balance between the limited amount of radiant energy received from solar radiation and general atmospheric and oceanic circulation, and the high proportion of that heat which is reflected back into space or otherwise tied up with in changes of phase of surface moisture. The net result is a permanent presence of ice and snow.

Climatologists have been concerned for some time about the possible climatic ramifications of increased atmospheric concentrations of anthropogenically produced pollutants. It now appears that due to progressive accumulation of greenhouse gases in the atmosphere, the global average surface temperature could rise between 1.5°C and 4.5°C above the mean 1980s decadal temperature within the next fifty years (IPCC, 1996). In high northern latitudes, the average rise in winter temperature is expected to be more than twice the global average.

A common hypothesis is that the most direct effect of global warming on the climate of the Arctic would be manifested in changes of the ice cover (Hare, 1982). Recent studies on the climatic role of sea ice do show the establishment of a bi-directional feedback mechanism between the ice and the adjacent atmospheric and oceanic systems (Walsh and Johnson, 1979; Parkinson, 1989; LeDrew et al., 1991). That is, sea ice has a significant influence on the atmosphere and adjacent oceans and in turn is significantly influenced by the atmosphere and oceans. There is now little doubt about the intimate connection between sea ice and climate and since the atmosphere, oceans, and ice cover are all components of a single thermodynamic system, a change in any one part necessarily results in compensating changes in the others (Maykut and Untersteiner, 1971; LeDrew, 1992, Fitzharris, 1996).

In areal extent, sea ice accounts for nearly two-thirds of the earth's ice cover. In the north polar region, it spans an area ranging from a summer minimum of 8.5 x 10^6 km^2 to almost twice that extent in winter. Although Arctic ice concentrations show considerable interannual variation (e.g. Chapman and Walsh, 1991; Piwowar and LeDrew, 1995) it is commonly held that they are predictable from month-to-month. That is, the ice concentrations for any month can be reliably estimated from the conditions present in the preceding month. This is based on our geophysical expectations that the thermal inertia of the ice pack will give it some month-to-month persistence. This tenet has never been tested statistically, however.
In this paper, we apply AutoRegressive-Moving Average (ARMA) statistical time series models to analyze the characteristics of the Northern Hemisphere sea ice cover as observed over a nine-year period. Our objective is not to identify climatically significant changes, rather it is to document the autoregressive nature of the ice cover so that statistically valid change analyses can be made in the future.

Time series modelling is a powerful tool for identifying temporal processes. The technique forces a critical evaluation of the data being examined based on a sound understanding of the phenomenon being modelled and also an appreciation of the mathematical attributes and limitations of the models being considered (Hipel and McLeod, 1994). For example, we expect to see seasonal variations in these time series because of planetary factors, and we expect to see autocorrelations in the first few lags because of sea ice persistence.

Also, the process of time series modelling imposes a detailed examination of the temporal structure of the data being modelled. The order of a statistically valid model, for example, may lead to further understanding of the degree of anomaly persistence in a region. Or, hidden climatic shifts may be revealed as non-stationarity in the model’s residual series.

Time series modelling is a robust mechanism for forecasting and simulation. Statistically based forecasts are necessary for continued resource management in the Arctic. Although current ice forecasting procedures have limited temporal range and possess significant errors (Chapman and Walsh, 1991), valid time series models have the potential to extend that range and add statistical significance levels to the predicted values. Further, sea ice simulations have the potential to provide valuable input to climate and engineering models by defining the spatial distribution of sea ice concentrations as a probability surface.

We begin our discussion with a description of the passive microwave satellite data we used and an overview of statistical time series analysis. We then present the results of fitting time series models across the Arctic sea ice zone highlighting regional anomalies arising from the analyses.

DATA

In this study, nine years of sea ice concentration data derived from Scanning Multichannel Microwave Radiometer (SMMR) imagery were analyzed. The SMMR data used in this study are derived from those published on the *Nimbus-7 SMMR Radiances and Sea Ice Concentrations* CD-
ROM (Volume 7) produced by the NASA Oceans and Ice Branch at the Goddard Space Flight Center and distributed by the National Snow and Ice Data Centre in Boulder, Colorado. Ice concentration is the percentage of a given area of ocean (a pixel, in this analysis) which is covered by ice and is a measure of the amount of open water within the ice margins. Even in the middle of winter, concentrations of less than 90% can be found in the middle of the Arctic pack ice largely as a result of open leads and polynyas. The SMMR data were resampled to a polar stereographic projection with a grid spacing of 25 km (Gloersen et al., 1993).

We analyzed 108 monthly average images acquired between November 1978 and August 1987. Although longer passive microwave time series are available, the SMMR data were ideal for our purposes since they came from a single sensor and time series models are very sensitive to external changes in state (such as might be introduced between successive sensors in a satellite series). Further, these data met the necessary requirements for hypertemporal analysis (Piwowar et al., 1997): (i) they are univariate at each temporal instance; (ii) they have precise co-registration of each time slice; and (iii) extraneous noise has been filtered out of each image. We reasoned that the knowledge gained from analyzing this controlled data set would give us greater confidence applying it to a more heterogeneous image source in the future.

**TIME SERIES ANALYSIS**

Time series analysis (TSA), the procedure of fitting a stochastic model to a given time series, is a multi-step process. The first stage of model construction is the description of the various statistical components of a time series through a variety of graphical and statistical techniques. Such properties as periodicity (monthly, seasonal, annual, etc.), trends, changes in mean (stationarity), and changes in variance (heteroscedasticity) are identified. This description of the properties of a given series is not only necessary to the identification of the proper model to be fit to the data but it also provides modellers with a basic appreciation of the complexities of their data.

Once a model with a potentially good statistical fit has been identified, the data are run through that model and a series of diagnostic tests are applied to the model's residual series to confirm or nullify its appropriateness. Since the residuals are the differences between modelled and observed values, they give an indication of how well the model has captured the characteristics of the process from which the original time series was created. One of the key diagnostics is to examine the residual series for autocorrelation. If there is any autocorrelation remaining in the residual series, then the model must be rejected.
ARMA Models

In this analysis of temporal variations in Arctic sea ice concentration, the class of linear time series models known as ARMA (AutoRegressive - Moving Average) models is considered (Box and Jenkins, 1976). Although there are other types of time series models which are used in a variety of other applications, Hipel and McLeod (1994) justify the use of ARMA class models for describing hydrologic and other kinds of environmental data sets. They show that, in addition to having a sound physical basis, ARMA models have the ability to forecast environmental time series at least as well as, and usually better than, their competitors.

In an autoregressive (AR) process, an observation is modelled as a function of previous observations' values. For example, if we were examining a time series of mean monthly temperatures and it was determined that this series was due to an AR process, then we would expect that a month which was warmer (cooler) than average would be followed by one or two further months which were also warmer (cooler) than their respective averages. If, on the other hand, we found that the present month's temperature depended less on a previous month's value and more on how much that previous month's value differed from its average value according to the model (i.e. the previous period's error term), then the series would be said to follow a moving average (MA) process. ARMA models can contain just AR components, just MA components, or combinations of the two.

Following the accepted nomenclature for these processes, a purely autoregressive model of order $p$ could be specified as ARMA($p,0$), or simply AR($p$). Mathematically, we write this as

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \cdots + \phi_p z_{t-p} + a_t$$

Equation [1] is known as an AR process of order $p$, or simply AR($p$), where the values $[\phi_1, \phi_2, \ldots \phi_p]$ are the AR parameters. Similarly, the notation ARMA($0,q$) or MA($q$) denotes a MA process of order $q$, given by:

$$z_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \cdots - \theta_q a_{t-q}$$

where $[\theta_1, \theta_2, \ldots \theta_q]$ are the MA parameters. ARMA($p,q$) references a mixed autoregressive - moving average process of the specified orders.

ARMA models are mathematical descriptions of the relationships between the discrete observations in a time series. That is, a given observation is defined as a function of one or more previous observations (for an AR process) and/or one or more previous observation's residuals (for
an MA process). As might be expected for data which are strongly influenced by planetary factors, the strongest relationships in the monthly SMMR data occur for observations which are 12 months apart. ARMA models were fit to the data only after this inherent seasonality was removed by subtracting each monthly image from its corresponding long-term (i.e., nine-year) monthly average. Figure 1 shows seasonal and deseasonalized time series for a location in Hudson Bay.

**TIME SERIES MODELLING OF THE SMMR DATA SET**

Time series modelling is typically a manual process, based on heuristic evaluation of graphical displays and summary statistics at each step of the modelling procedure. One of our principal objectives was to analyze the entire SMMR data set on a per-pixel basis. We recognized that this would be impractical if done manually so we would need to automate the modelling procedure as much as possible and that this would require some basic assumptions about the data. The most critical assumption was the hypothesis that most of the Arctic ice concentration time series could be fit with ARMA models of the same degree. Consequently we attempted to determine if a single time series model might be widely applicable by manually fitting ARMA models to twelve time series sampled from diverse parts of the Arctic sea ice zone (labelled "Original Data Locations" in Figure 2). The preliminary results (presented in Table 1) suggested that most of the region's sea ice concentrations might be adequately characterized by AR(1) models. Since first-order autoregressive models were consistent with our geophysical assumptions regarding month-to-month sea ice persistence, we proceeded to fit AR(1) models to the entire deseasonalized SMMR data set on a per-pixel basis.¹

**RESULTS**

There were 136,192 potential time series (i.e., pixels) in the sea ice concentration images. The ARMA modelling process fails, however, when there is minimal variability in a time series. This was the case over the land and open ocean region that had no recorded ice concentration values at any time. This left only 61,513 time series (including all of the Arctic sea ice zone) for which AR(1) models were actually calculated.

¹ Complete time series model fitting details are given in Piwowar and LeDrew (2001).
A tally of the spatial coverage of the Arctic sea ice zone which passes all of the TSA diagnostic criteria reveals that only 62% of the time series are adequately fit by an AR(1) model, as shown in white on Figure 3. For these locations, we can say that the mean monthly sea ice concentrations can be estimated with statistical confidence by knowing the long-term average concentration for that month and the previous month's value at that location. This statistical analysis matches the geophysical reality of the Earth's sea ice cover by noting that an annual cycle is induced on the cryosphere by planetary factors and that the thermal properties of ice make it relatively persistent with respect to adjacent atmospheric and oceanic influences.

**Non-AR(1) Regions**

An important issue is the remaining 38% of the Arctic sea ice zone which does not match our geophysical expectations. These areas are dispersed throughout the Arctic, but have higher concentrations in portions of the Beaufort Sea, southern Hudson Bay, the Sea of Okhotsk, the East Siberian Sea, the Barents Sea, and the Labrador Sea (as shown in black on Figure 3).

Failure of first-order autoregressive models to characterize the monthly time series (the black areas in Figure 3) could be attributed to insignificance of the AR(1) parameters and autocorrelation in the residual series. These factors are shown spatially in Figure 4. Each image in Figure 4 is coded so that darker areas represent greater deviation from a valid AR(1) model. Thus the whiter the region, the stronger the AR(1) model is. For example, in Figure 4a higher parameter estimates (whiter pixels) represent stronger month-to-month correlations. Further, the more significant\(^2\) the parameter estimates are (whiter pixels in Figure 4b), the more confidence we can place in the tested model. Finally, lower residual variances (whiter pixels in Figure 4c) are indicative of statistically better fitting models.

Using the information from Figure 4 as a guide, six new time series were extracted at locations where the monthly sea ice concentrations were not well-fit by an AR(1) model. These additional data locations are identified in Figure 2. The same ARMA models which were used to evaluate the initial 12 sample time series were initially applied to the data from the six new locations. Additional models containing higher-order parameters (e.g. AR(6)) and constrained parameters (i.e. specifically setting some parameters to zero) were then applied to the data until a satisfactory process was identified. Every model that was tested was evaluated with reference to the same

\(^2\) Significance was measured with the \(t\) statistic.
diagnostics described above: significance of the model's parameters and independence in the residual series. These results are summarized in Table 2.

In every case, the accepted models in Table 2 are a noticeable departure from the previous AR(1) models which were fit to nearby locations within the same geographic region. By looking at intra-regional differences in the time series plots for each regional pair (Figure 5) the different modelling needs for each series are quite evident for some locations (e.g. Labrador and Barents Seas) yet imperceptible at other places (e.g. East Siberian Sea and Hudson Bay). Of note is the inclusion of AR parameters at lags 6 or 12 in four of the series indicating significant residual seasonality not accounted for by the basic AR(1) models. Three of the new series also benefited from the addition of MA parameters to the model attesting to a degree of variability in the data which is not well accommodated by a purely autoregressive model. By assimilating important details about these regions from the time series analyses geophysical interpretations for each of the six regions are given below.

**Arctic Ocean**

The AR(1) parameters are statistically significant over almost all of the Arctic sea ice zone (Figure 4* b*). The highest significance levels are found in the central Arctic Basin, north of the Canadian Archipelago. Although this implies a high degree of confidence in the AR(1) models fit to these data, a review of the diagnostic residual checks revealed some locations in this region where there is a poor fit, mostly because of significant residual autocorrelations at lag 2, indicating a longer temporal persistence of the ice features and a need for more AR parameters. This is substantiated by modelling at a nearby location in the Beaufort Sea where an AR(6) model (with the AR(3-5) parameters constrained to zero) was found to be appropriate (Table 2). Thus the ice concentrations in this area depend not only on that of the previous month, but also on values from the second and sixth previous months. Further, for pixels in this region the AR(1) parameters themselves are relatively high, indicating an elevated degree of correlation between months (Figure 4* a*). These observations fit the geophysical reality that this region is the domain of the least amount of ice motion in the Arctic leading to very little change in ice concentrations from month to month, or even from season to season.

**East Siberian Sea**

Another area of notably high and highly significant AR(1) parameters is in the East Siberian Sea (Figures 4* a*,* b*). Associated with the regions of parameter significance, however, are pockets of
elevated model variances and significant autocorrelations at lags 2 and 24 in the residual series (Figure 4c). When a new time series model was fit to some of these data, the resulting AR(12) model (with the AR(3-11) parameters constrained to zero) does much better in explaining the strong monthly and seasonal autocorrelations in these data (Table 2). The seasonal AR(12) parameter of the fit model and the significant lag 24 autocorrelations arising from the original series are harder to explain, however, given that the time series had been deseasonalized prior to modelling. Unlike the Beaufort Sea data, stagnant ice cannot be used to justify these findings since this area is located near the origin of the Transpolar Drift Stream and has a mean annual ice motion of approximately 2 cm/sec (about average for the Arctic Basin). These "seasonal" autocorrelations may be the result of the quasi-biennial oscillation of the polar ice pack that has been observed to shift between the North American and Eurasian coastlines (Gloersen et al., 1993). The irregularity of this phenomenon would allow it to remain in the time series even after the regular seasonal cycle had been removed.

**Labrador Sea**

An interesting dichotomy is revealed in the Davis Strait - Labrador Sea region, where the sea ice features in the western half strongly follow an AR(1) process, but the temporal characteristics of the ice in the eastern portion strongly contradict the AR(1) models fit there. The western section is characterized by AR(1) models whose residual series have very low variances and no significant autocorrelations, both indicative of a good statistical fit (Figure 4c). The poor fit in the east, however, is defined by very high variances and many significant autocorrelations in the residual series. Interestingly, the strength and significance of the AR(1) parameters are about the same in both sections, yet peak at a ridge dividing the two areas (Figures 4a,b). To examine this phenomenon in greater detail, three time series were extracted from the data to show the annual cycle of ice concentrations at locations in the western region, eastern region, and at a point in the middle. These are plotted in Figure 6. It is evident that all three series are in the seasonal sea ice zone, but they have little else in common. The very regular nature of the western time series attests to its match to an AR(1) process. The series from the central location is located along the white ridge seen on the AR(1) parameter image (Figure 4a) and was determined to be best fit by an ARMA(1,1) model. The eastern location is evidently at the margin of the seasonal sea ice zone, experiencing very little winter iciness over the SMMR data record, except during the first six months of 1983 and 1984, when ice concentrations in this region exceeded 80%. (Anomalous ice conditions across the Arctic were prevalent during this period, likely the result of a particularly strong El Niño event during 1982-83). During the selected model re-fitting described above, this time series was determined to be adequately modelled by an ARMA(12,2) process with the
AR(1–11) parameters constrained to 0 (Table 2). These sea ice anomalies are explained by reviewing the ocean currents in this region. The western half of the Davis Strait - Labrador Sea region is dominated by the southerly Labrador Current which is responsible for carrying much of the sea ice which is exported from the Canadian Archipelago southward. In the eastern half, warmer water is carried northward along the West Greenland Current. Evidently in years of very heavy ice conditions across the Canadian Arctic, such as in 1983 and 1984, the volume of southerly ice export along the Labrador Current increases and is forced to spill over into the waters typically occupied by the West Greenland Current producing the heavy ice anomalies observed here. This has also been observed by Gloersen et al., (1993).

**Sea of Okhotsk and Barents Sea**

Another interesting observation which can be made from the three Labrador Sea time series shown above is the change from a pure AR process for the (western) series in the seasonal sea ice zone, through a mixed ARMA model along the sea ice margin, and to an almost pure MA process at a point (in the eastern Labrador Sea) which can be considered to be outside of the seasonal sea ice zone. This would suggest that as the seasonal cycle of ice concentrations becomes more irregular, its autoregressive nature is gradually replaced by a stronger relationship with past months' residuals than their observations. This observation is substantiated by paired time series extracted from the Sea of Okhotsk and the Barents Sea (Figure 5). In each region, pure AR processes were appropriate for the series farther into the ice pack while MA parameters were more dominant at locations closer to the ice edge (Table 2).

**Hudson Bay**

In contrast to the preceding interpretations, there is a region in southern Hudson Bay for which the AR(1) parameter values and their significances are low (Figures 4a,b). A review of the modelling results shows that many of the residual series in this region have multiple significant autocorrelations, generally beginning at lag 2. In a comparison of the time series plots of the sea ice concentration anomalies for a point in this region and for a nearby Hudson Bay position where an AR(1) model is valid, the overall nature of the anomalies is similar but the latter series is seen to be more consistent (Figure 7). During an extensive modelling exercise in which thirty models were compared, the best model which could be fit to the anomalous region was an ARMA(2,10) process with the first eight MA parameters constrained to 0. While the inclusion of the AR(2) parameter is
indicated by the large lag 2 residual autocorrelation from the original AR(1) model, the apparent need for high-order MA parameters cannot be explained in the geophysical context of sea ice concentrations in Hudson Bay. This situation illustrates the condition where the statistically best model may not be the most appropriate model in reality if it has no explainable physical justification (Gottman, 1981). Therefore the temporal nature of the sea ice processes in southern Hudson Bay remain an enigma. Neighbouring models and knowledge of the geophysical nature of sea ice suggest that these processes should be AR(1) or AR(2), although this cannot be statistically demonstrated. We note that other attempts to model the Hudson Bay ice cover have also been challenged by its variability (e.g., Wang et al., 1994).

**CONCLUSIONS**

Implicit in many studies of Arctic sea ice processes is the assumption that most Arctic sea ice concentrations are heavily dependent on the previous month's conditions and the period in which the measurements are made. In this paper we have rigorously evaluated this assumption with statistical time series models. ARMA modelling of a deseasonalized passive microwave data set revealed that only 62% of the Arctic sea ice zone meets this assumption. Although our analyses were limited to a nine-year time series, we believe that our findings would not differ significantly when data from a longer period are examined.

We have shown that much of the variability in concentrations of Arctic sea ice do follow an AR(1) process. Sea ice dynamics is based on a complex mixture of geophysical forces, such as incident solar radiation, atmospheric and oceanic currents, ice structure, growth and decay rates, thermal properties, and thickness distributions, to name a few. Many of these forces operate at different temporal scales than at the monthly interval in which we examined the sea ice record. The influences from passing atmospheric weather systems, for example, are relatively short-lived and averaged-out of the monthly composites. Conversely, the effects that more global changes in atmospheric systems, such as El Niño, have are on the temporal order of several months to one year. We believe that the effectiveness of the AR(1) model is largely due to the thermal inertia of the ice cover. Sea ice is a mixture of four components: ice, liquid brine, air bubbles, and solid salts (Wadhams, 2000). Although the conduction of heat through sea ice is influenced by all of these components, the most complex effect occurs with liquid brine. The liquid brine acts as a thermal reservoir, absorbing and releasing latent heat in the ice as the internal temperatures change, thus adding extra resistance to warming and cooling (Wadhams, 2000). This gives the ice cover the inter-month persistence which is identified by the AR(1) models.
The distribution of valid AR(1) models is not consistent across different regions within the Arctic sea ice zone. As might be expected, the strongest autoregressive correlations are found in the central Arctic Basin and generally become weaker closer to the southern limit of the seasonal ice extent. As the autoregressive nature of the temporal processes decreases in a southward progression, the mean ice concentration for a given month is shown to become less dependent on past months' values and more related to previous observations' residuals, i.e. the processes are increasingly more appropriately fit by moving average models. This may have a geophysical basis since the presence of sea ice becomes much more irregular along the lower bounds of the seasonal sea ice zone.

Even within the same geographic region, temporal sea ice processes can be clearly different. Our analyses showed that neighbouring locations, less than 100 km apart, often required quite different models, highlighting the spatial variability of temporal ice concentration patterns and also demonstrating the sensitivity of the ARMA modelling process.

The results presented above have important implications for our understanding of the month-to-month changes in Arctic sea ice concentrations. While over 60% of the region does meet our geophysical expectation that there would be strong month-to-month persistence (due to the thermal inertia of the ice pack), a large proportion of the area appears to contradict this prospect. These are the regions where other factors dominate and add unpredictability to the monthly progression. For example, large stationary weather systems have been shown to reverse the normal motion of the ice pack in the Beaufort Sea (Serreze et al., 1989; LeDrew et al., 1991) or to shift the entire polar ice pack between the North American and Eurasian coastlines (Gloersen et al., 1993). Another source for the irregularity could be abrupt changes in ice concentrations brought on by global shifts in weather patterns, such as the strong El Niño event which occurred in 1983 in the middle of the SMMR time series. A third source of variability arises from changes seasonal along the marginal ice zone. This area, with its tenuous ice cover, can rapidly change from being totally ice covered to completely open ocean with the passage of smaller storm systems. Although we cannot determine the exact causes of the observed non-autoregressive monthly changes in ice concentrations, our results do tell us that there are some regions which are more susceptible to high variability (hence more difficult to model) than others.

Time series analyses such as these are of potential importance in ice forecasting. Current ice forecasting procedures are limited to only about one week and possess significant errors even at that range (Chapman and Walsh, 1991). Statistically based forecasts are necessary for continued
resource development in the Arctic. In Piwowar and LeDrew (2001) we demonstrate the robustness of the autoregressive models by fitting them to a split sample of our time series and comparing the forecast and observed ice concentrations with favourable results.

The results presented above have important implications for Arctic sea ice modelling. While most of the Arctic does follow an AR(1) process, clearly it is inappropriate to assume strong month-to-month correlation in ice concentrations are occurring at any given location. There is a pressing need to examine, in greater detail, the nature and causes of non-autoregressive monthly ice concentrations and to compare these findings with time series derived from other temporal data sets.

**ACKNOWLEDGEMENTS**

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**REFERENCES**


Figure 1: Mean Monthly Sea Ice Concentrations for a Location in Hudson Bay.
Figure 2: Location Map for Time Series Selected for Detailed Modelling
Figure 3: Spatial Distribution of Accepted AR(1) Models
These results are derived from a mathematical combination of the data shown Figure 4.
Figure 4: AR(1) Modelling Results
Figure 5: Time Series Plots of Neighbouring Processes
Figure 6: Seasonal Time Series Plots for Three Locations in the Labrador Sea
Figure 7: Deseasonalized Time Series Plots for Neighbouring Locations in Hudson Bay
Table 1: Most Appropriate Models Fit to the Original Twelve Time Series

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<thead>
<tr>
<th>Series Location</th>
<th>ARMA Model</th>
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<tr>
<td>Sea of Okhotsk</td>
<td>AR(1)</td>
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<td>ARMA(1,1)</td>
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<td>Newfoundland</td>
<td>AR(1)</td>
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<td>Barents Sea</td>
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<td>Bering Sea</td>
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<td>Kara Sea</td>
<td>AR(1) or MA(1)</td>
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<td>Hudson Bay</td>
<td>AR(1) or MA(1)</td>
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<td>Beaufort Sea</td>
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Table 2: Most Appropriate Models Re-Fit to Selected Time Series

<table>
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<tr>
<th>Series Location</th>
<th>ARMA Model</th>
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<td>ARMA(12,2) model with AR(1-11) parameters constrained to 0</td>
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<tr>
<td>Labrador Sea</td>
<td>ARMA(12,2) model with AR(1-11) parameters constrained to 0</td>
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<td>Barents Sea</td>
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<td>ARMA(2,10) model with MA(1-8) parameters constrained to 0</td>
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<td>Beaufort Sea</td>
<td>AR(6) model with AR(3-5) parameters constrained to 0</td>
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