Koper Summer School - Problem Set Day 4

Further Topics

1. Learn about "orbital association schemes". Let $H \leq G$, then the action of G on the pairs of cosets forms an association scheme if and only if $H \leq G$ is multiplicity-free.

2. The Johnson scheme is the orbital association scheme for $G = \mathrm{Sym}(n)$ and $H = \mathrm{Sym}(k) \times \mathrm{Sym}(n-k)$.

3. The **perfect matching association scheme** is the orbital association scheme for $G = \mathrm{Sym}(2k)$ and $H = \mathrm{Sym}(2) \wr \mathrm{Sym}(k)$. Learn about this scheme.

4. Check out Wilson's 1984 paper with the proof of the exact bound in the t-intersecting version of EKR.
   See:
   Wilson, Richard; The exact bound in the Erdős-Ko-Rado theorem. Combinatorica 4 (1984), no.2-3, 247–257.

5. Check out Frankl and Wilson's paper with the proof of the exact bound in the t-intersecting version of EKR for the q-Kneser graphs.
   See:
   P. Frankl and R. M. Wilson. The Erdős-Ko-Rado theorem for vector spaces. J. Combin. Theory Ser. A, 43(2):228–236, 1986.

   (Note how the calculations are easier for the q-Kneser than the Kneser.

6. Check out Godsil and Guo's paper describing Wilson's matrix:
   Using the existence of t-designs to prove Erdős-Ko-Rado

7. Learn about the complete EKR theorem - this result describes the largest collect of t-intersecting sets when n is small relative to k and t. These were proven by Ahlswede and Khatachatrian, and I don't know if there is an algebraic proof!
   See:
   Ahlswede, Rudolf; Khachatrian, Levon H. The complete intersection theorem for systems of finite sets. European J. Combin.18(1997), no.2, 125–136.