

Koper Summer School - Problem Set Day 3

1. Derangements

- (a) Give a recursive formula for the number of derangements in $\text{Sym}(n)$.
- (b) Show that the number of derangements in $\text{Sym}(n)$ is close to $\frac{n!}{e}$.
- (c) Try calculating the eigenvalues of Γ_G for some small groups G . (Any regular group, a Frobenius group, $\text{Sym}(4)$ for some values of q , or all values).
- (d) If G is a 2-transitive group, let $\chi(g) = \text{fix}(g) - \mathbf{1}$. Show that the χ -module is spanned by the balanced characteristic vectors of the canonical intersecting sets $S_{i,j}$.
- (e) Find the eigenvalues of the derangement graph of $\text{PGL}(2, q)$.
- (f) All the dihedral groups have the property the only the canonical intersecting set are maximum intersecting set. Prove this by looking at the group action.

2. Cayley graphs

- (a) Show a Cayley graph $\text{Cay}(G, C)$ is connected if and only if C generates the group.
- (b) When is $\text{Cay}(G, C)$ the join of smaller graphs?

3. Clique-coclique bound

- (a) Apply the clique-coclique bound for Hamming graph? Can you find all cocliques in this graph using the clique-coclique bound.
- (b) Apply the clique-coclique to the Kneser graphs. When does this bound hold with equality?
- (c) Read Wang and Zhang's paper proving uniqueness of the maximum cocliques in $\Gamma_{\text{Sym}(n)}$ using the clique-coclique bound.