University of Regina
Department of Mathematics and Statistics

MATH431 – Differential Geometry – Winter 2014

Homework Assignment No. 1

1. Find the curvature of the ellipse at an arbitrary point (see the notes, Section 1.2, Example 3) and check that if $a > b$ then the ellipse is more curved at $(a, 0)$ than at $(0, b)$.
2. In the context of section 1.5 of the notes, check that the osculating circle given by Definition 1.5.2 satisfies the conditions (i), (ii), and (iii).
3. Show that the evolute of the ellipse
$$\alpha(t) = (a\cos t, b\sin t)$$
is the curve of equation[1]
$$\bar{\alpha}(t) = \left(\frac{(a^2 - b^2)\cos^3 t}{a}, \frac{(b^2 - a^2)\sin^3 t}{b}\right).$$
4. (a) In the context of section 1.6 of the notes, find a parametrization of the cycloid (as $\alpha(t) = (x(t), y(t))$) taking the radius of the circle to be 1. **Hint:**You may want to assume that the circle rolls along the x axis and the initial position of the point on the circle is the origin O (see Figure 1 below). Choose the parameter t as the angle between the line segments $C\alpha(t)$ and CA.
   (b) Find all points t with $\alpha'(t) = 0$ (these are called *singular* points).
   (c) Determine the length of the piece of the cycloid which corresponds to a complete (that is, of 360^0) rotation of the circle.
   (d) Find the limits of the slope of the tangent line to the cycloid at $\alpha(t)$ as $t \to 2\pi$, $t < 2\pi$, respectively $t \to 2\pi$, $t > 2\pi$.
5. In the context of section 1.6 of the notes, find a parametrization of the cardioid (as $\alpha(t) = (x(t), y(t))$) taking the radius of the two circles to be 1. Then find the curvature of the cardioid at an arbitrary point.
6. Find a parametrization by arc length of the catenary (see Section 1.6 of the notes). For simplicity, take $a = 1$.
7. Show that for any point P on the tractrix (see Section 1.6 of the notes) the length of the tangent line segment between P and the y axis is constant (independent of t).

[1] See section 1.5 of the notes for a figure.

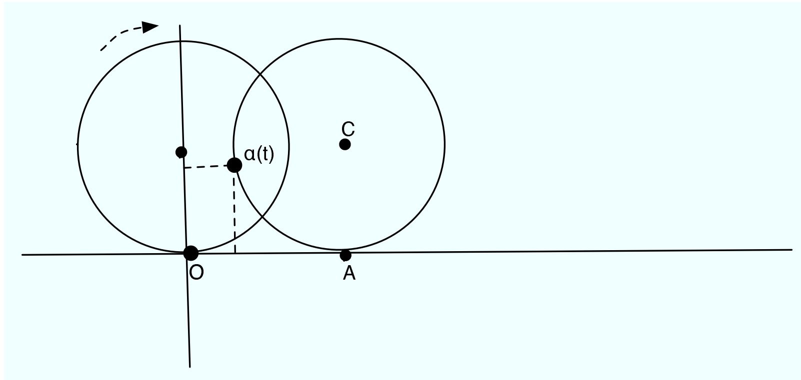

Figure 1