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Violation analysis for solid waste management systems: an interval fuzzy programming approach

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This paper introduces a violation analysis approach for the planning of regional solid waste management systems under uncertainty, based on an interval-parameter fuzzy integer programming (IPFIP) model. In this approach, several given levels of tolerable violation for system constraints are permitted. This is realized through a relaxation of the critical constraints using violation variables, such that the model's decision space can be expanded. Thus, solutions from the violation analysis will not necessarily satisfy all of the model's original constraints. Application of the developed methodology to the planning of a waste management system indicates that reasonable solutions can be generated through this approach. Considerable information regarding decisions of facility expansion and waste flow allocation within the waste management system were generated. The modeling results help to generate a number of decision alternatives under various system conditions, allowing for more in-depth analyses of tradeoffs between environmental and economic objectives as well as those between system optimality and reliability.

Keywords: decision-making, environment, fuzzy programming, interval, optimization, planning, regret analysis, solid waste, uncertainty.

Introduction

Municipal solid wastes (MSW) can be recycled, composted, incinerated or landfilled. No matter which method is used, transport of MSW to waste management facilities is needed. The costs for waste transportation and disposal consume major fractions of operating budgets for municipal waste management systems. Identification of desirable waste-flow-allocation patterns is thus an important aspect of municipal solid waste management planning.

In MSW management systems, there are many processes that should be considered by decision-makers, such as waste collection, transportation, treatment and disposal (Wilson, 1985). These processes are related to a multitude of impact factors that interact with each other with multi-period, multi-layer, and multi-objective features (Thomas *et al.*, 1990). Thus, for MSW managers who are

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facing conflicts of meeting required environmental objectives and minimizing system costs, systems analysis techniques may be particularly helpful for making tradeoffs and yielding optimal decision schemes (Baetz, 1990a,b; Thomas *et al*., 1990).

Since the 1960s, various mathematical programming models have been developed for supporting decisions of MSW managers and evaluating relevant operational and investment policies (Kirca and Erkip, 1988; Zhu and Revelle, 1990; Chang and Wang, 1995; Chang *et al*., 1997). The problems addressed included waste flow allocation, technology screening, as well as siting, sizing and timing of system expansions (Gottinger, 1986; Chang and Wang, 1993 and 1994; Chang and Lu, 1997). For example, Anderson (1968) proposed an economic optimization method for the planning of MSW management systems in the 1960s; Hsieh and Ho (1993) and Lund and Tchobanoglous (1994) used linear programming (LP) to optimize solid waste disposal and recycling systems.

However, with increasing waste generation, the reduction of available land, and the emerging technologies of waste recycling and composting, municipal solid waste management systems are becoming more and more sophisticated. Many system parameters, impact factors and their interactions are associated with uncertainties (Huang *et al*., 1992, 1993a,b and 1995; Chang *et al*., 1997). Various methods dealing with uncertainty have been developed for planning solid waste management systems, such as fuzzy mathematical programming (FMP) (Leimbach, 1996; Chang *et al*., 1997a; Chanas and Zielinski, 2000), stochastic mathematical programming (SMP), and interval mathematical programming (IMP) (Huang *et al*., 1992, 1993a,b and 1995; Chang *et al*., 1997b). There are two major FMP approaches: possibilistic programming and flexibility programming. Their major problems include (i) possibilistic FMP may lead to complicated sub-models which may not be applicable to practical problems (Alberti and Perrone, 1999; Tonon and Bernardini, 1999), and (ii) flexible FMP is unable to communicate some uncertain constraints directly into the optimization process (Huang *et al*., 1993b). As for SMP, the major problem is its requirement for detailed probability distributions that may be difficult to obtain. For example, a planner may know that the daily waste generation rate fluctuates within a certain interval, but he/she may find it difficult to state a reliable probability distribution for this variation (Ermoliev and West, 1988; Jansson, 1988; Marti, 1990). More recently, Huang *et al*. (1993a) developed interactive IMP approaches and applied these to practical problems. In these methods, interval numbers were directly incorporated into the optimization process, with the solutions presented as stable intervals. Combining the advantages of IMP and FMP, interval-fuzzy programs (IFMP) have been proposed to model uncertain systems that contain both interval and fuzzy information (Huang *et al*., 1993b and 1995).

However, IMP models may become infeasible when some right- and/or left-hand side parameters have large intervals. This occurs when some system conditions are very uncertain, making it difficult to quantify these possible ranges as reasonably small intervals. Also, decision-makers may desire tradeoff information between system benefits and reliability. This can only be obtained through solving the model under different levels of system constraint violation and analyzing the generated alternatives. Existing IMP approaches cannot facilitate such violation analyses, since the model will be simply infeasible when any of its constraints is violated (Huang *et al*., 1995; Ellis *et al*., 1996; Huang, 1998).

As an extension to the existing modeling approaches, an interval fuzzy violation analysis approach based on methods of IFMP and regret analysis (Ellis, 1988; Burn, 1992; Ellis *et al*., 1994 and 1996) is developed in this paper to address the above deficiencies. In this method, several given levels of tolerable violation for system constraints are permitted. This is realized through relaxing the critical constraints using violation variables, such that the model's decision space can be expanded. Thus, solutions from the violation analysis will not necessarily satisfy all of the model's original constraints. This will help to generate a range of decision alternatives under various system conditions, allowing more in-depth analyses of tradeoffs between environmental and economic objectives as well as those between system optimality and reliability. The method is applied to the planning of a hypothetical municipal solid-waste management system to demonstrate its potential usefulness.

Methodology

Consider an interval-parameter linear program (ILP) as follows:

$$\text{Min } f^{\pm} = C^{\pm}X^{\pm} \quad (1a)$$

Subject to

$$A^{\pm}X^{\pm} \leq B^{\pm} \quad (1b)$$

$$X^{\pm} \geq 0 \tag{1c}$$

where $A^{\pm} \in \{R^{\pm}\}^{m \times n}$, $B^{\pm} \in \{R^{\pm}\}^{m \times 1}$, $C^{\pm} \in \{R^{\pm}\}^{1 \times n}$ and $X^{\pm} \in \{R^{\pm}\}^{n \times 1}$ ($R^{\pm}$ denotes a set of interval numbers).

When the system's goal and constraints are fuzzy, model (1) can be converted into an IFMP problem, through incorporating fuzzy-programming concepts within the ILP framework (Huang *et al.*, 1993b and 1995). Thus, an interval-parameter fuzzy linear programming (IFLP) model can be formulated as follows (Huang *et al.*, 1993b):

$$\text{Max}\, \lambda^{\pm} \tag{2a}$$

Subject to

$$C^{\pm}X^{\pm} \leq f^{+}_{opt1} - \lambda^{\pm} \cdot \left[f^{+}_{opt1} - f^{-}_{opt1}\right] \tag{2b}$$

$$A^{\pm}X^{\pm} \leq B^{+} - \lambda^{\pm} \cdot [B^{+} - B^{-}] \tag{2c}$$

$$0 \leq \lambda^{\pm} \leq 1 \tag{2d}$$

where f^{-}_{opt1} and f^{+}_{opt1} denote the least and most desirable system objectives, respectively; and $\lambda^{\pm}$ is a control variable corresponding to the degree (membership grade) to which the model's solution fulfills the fuzzy goal or constraints. Specifically, flexibility in the constraints and fuzziness in the system objective, which are represented by fuzzy sets and denoted as 'fuzzy constraints' and a 'fuzzy goal', respectively, are expressed as membership grade $\lambda^{\pm}$ corresponding to the degrees of satisfaction for the constraints and the objective. An interactive two-step solution algorithm for solving model (2) was provided by Huang *et al.* (1993b). The resulting solution is presented as sets of intervals:

$$x^{\pm}_{j\,opt} = [x^{-}_{j\,opt}, x^{+}_{j\,opt}]$$

where $x^{\pm}_{j\,opt} \in X^{+}_{opt}$; $f^{\pm}_{opt} = [f^{-}_{opt}, f^{+}]$; and $\lambda^{\pm}_{opt} = [\lambda^{-}_{opt}, \lambda^{+}_{opt}]$.

However, simply providing the above modeling solution may not be sufficient for decision-makers. Instead, they may desire tradeoff information between system satisfaction (with low net cost) and system reliability (with low risk). This can only be obtained through solving the model under different risk levels of violating system constraints and analyzing the generated alternatives.

One potential alternative for dealing with this issue is to reconfigure the model's decision space through the introduction of a number of violation variables into model (2) to relax its constraints. At the same time, a tolerable violation level should be predefined. For example, for a solid waste management system, waste flow delivered to a facility could exceed its planned capacity to a certain extent, but should never be more than its design capacity (tolerable limit). Thus, to facilitate violation analysis, model (2) can be converted to:

$$\text{Max}\, \lambda^{-} \tag{3a}$$

Subject to

$$C^{+}X^{+} - V_f \leq f^{+}_{opt1} - \lambda^{-} \cdot \left[f^{+}_{opt1} - f^{-}_{opt1}\right] \tag{3b}$$

$$A^{-}X^{+} - V \leq B^{+} - \lambda^{-} \cdot [B^{+} - B^{-}] \tag{3c}$$

$$V_f + \sum_{l=1}^{m} V_l \leq \lambda^{-} \cdot TV \tag{3d}$$

$$0 \leq \lambda^{-} \leq 1 \tag{3e}$$

where V_f is a violation variable for the objective function; V is a set of violation variables for constraints, $V = (V_1, V_2, \ldots, V_m)$; TV is the total tolerable violation limit; and m is the number of constraints. Here, if V_f and V are different units, they should be normalized before they are added together. The simplest method is to divide V_f and V by the average of the lower bound and the upper bound of the objective or right-hand constraints, respectively. When $TV = 0$, model (3) is a submodel of a conventional IFLP problem (Huang *et al.*, 1993), where either the system goal or the constraints will be violated. When $TV > 0$, the original constraints in model (2) are allowed certain levels of violation, which are associated with relevant risk levels. Thus, with varied TV levels, solutions with different λ^{-} values will be generated. These solutions are useful for analyzing tradeoffs between the system-satisfaction levels and the associated risks (of violating the fuzzy goal and constraints). Model (3) can also be used for analyzing the sensitivity of the system objective to coefficients in the fuzzy goal and constraints, with the results being potentially helpful for improving cost effectiveness in managing a MSW system.

An illustrative example is now formulated to demonstrate applicability of the proposed violation-analysis method. First, consider an ILP model as follows:

$$\text{Max}\, f^{\pm} = [26,\, 30] \cdot x^{\pm}_1 - [5{\cdot}5,\, 6{\cdot}0] \cdot x^{\pm}_2 \tag{4a}$$

Subject to

$$[8, 10] \cdot x^{\pm}_1 - [12,\, 14] \cdot x^{\pm}_2 \leq [3{\cdot}8,\, 4{\cdot}2] \tag{4b}$$

$$[2{\cdot}4, 2{\cdot}8] \cdot x^{\pm}_1 + [3, 4] \cdot x^{\pm}_2 \leq [6{\cdot}0, 6{\cdot}5] \tag{4c}$$

$$x^{\pm}_1, x^{\pm}_2 \geq 0 \tag{4d}$$

According to Huang (1998), the above model can be converted into the following two submodels:

$$\text{Max } f^{+} = 30x_1^{+} - 5{\cdot}5x_2^{-} \quad (5a)$$

Subject to

$$8x_1^{+} - 14x_2^{-} \leq 4{\cdot}2 \quad (5b)$$

$$2{\cdot}4x_1^{+} + 4x_2^{-} \leq 6{\cdot}5 \quad (5c)$$

$$x_1^{+}, x_2^{-} \geq 0 \quad (5d)$$

and

$$\text{Max } f^{-} = 26x_1^{-} - 6{\cdot}0x_2^{+} \quad (6a)$$

Subject to

$$10x_1^{-} - 12x_2^{+} \leq 3{\cdot}8 \quad (6b)$$

$$2{\cdot}8x_1^{-} + 3x_2^{+} \leq 6{\cdot}0 \quad (6c)$$

$$x_1^{-} \leq x_{1opt1}^{+} \quad (6d)$$

$$x_2^{+} \geq x_{2opt1}^{-} \quad (6e)$$

$$x_1^{-}, x_2^{+} \geq 0 \quad (6f)$$

where x_{1opt1}^{+} and x_{2opt1}^{-} are solutions from submodel (5). Thus, the optimal solutions for ILP model (4) are: $x_{1opt1}^{\pm} = [1{\cdot}32, 1{\cdot}62], x_{2opt1}^{\pm} = [0{\cdot}64, 0{\cdot}73]$, and $f_{opt1}^{\pm} = [29{\cdot}94, 45{\cdot}08]$.

Based on (2) and Huang *et al.* (1993b), model (4) can be converted to an IFLP problem as follows:

$$\text{Max } \lambda^{\pm} \quad (7a)$$

Subject to

$$[26, 30] \cdot x_1^{\pm} - [5{\cdot}5, 6] \cdot x_2^{\pm} \geq f_{opt1}^{-} + \lambda^{\pm}[f_{opt1}^{+} - f_{opt1}^{-}] \quad (7b)$$

$$[8, 10] \cdot x_1^{\pm} - [12, 14] \cdot x_2^{\pm} \leq 4{\cdot}2 - 0{\cdot}4 \cdot \lambda^{\pm} \quad (7c)$$

$$[2{\cdot}4, 2{\cdot}8] \cdot x_1^{\pm} + [3, 4] \cdot x_2^{\pm} \leq 6{\cdot}5 - 0{\cdot}5 \cdot \lambda^{\pm} \quad (7d)$$

$$0 \leq \lambda^{\pm} \leq 1 \quad (7e)$$

According to Huang *et al.* (1993b), the solutions for IFLP model (7) are: $x_{1opt}^{\pm} = [1{\cdot}41, 1{\cdot}54]$, $x_{2opt}^{\pm} = [0{\cdot}60, 0{\cdot}83]$, and $\lambda^{\pm} = [0{\cdot}136, 0{\cdot}816]$. Figure 1 shows a graphical depiction of this problem. The decision space is divided by constraints (4b) and (4c) into eight zones (i.e. zones *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, and *I*). For example, any solution in zone A will be feasible since it always satisfies both constraints (4b) and (4c); in comparison, any point in zone *I* will be infeasible. A more complicated situation is for points in zones *B*, *D*, *E*, *F* and *H*, where a term 'softly feasible' is used when the solution feasibility is dependent upon the detailed location of the decision variables and the conditions of the uncertain constraints. In fact, this 'soft' concept

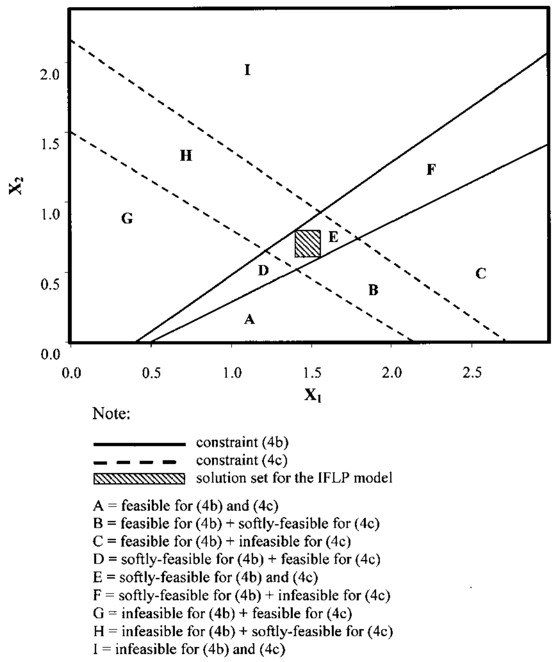

Figure 1. Inexact constraints and solution set for the IFLP model.

corresponds to the existence of uncertainties, reflecting the fact that there exists a trade-off between system benefit and risk of violating the constraints. A high benefit will correspond to a high cost, and we cannot get both high benefit and low risk at the same time, due to the effect of uncertainties. For the study problem, the solution set for the two decision variables is presented as a rectangle located in the zone being softly feasible for both constraints (i.e. zone *E*).

For further violation analysis, a submodel based on IFLP model (7) can be formulated as follows (Huang *et al.*, 1995):

$$\text{Max } \lambda^{-} \quad (8a)$$

Subject to

$$26 \cdot x_1^{-} - 6 \cdot x_2^{+} \geq f_{opt1}^{-} + \lambda^{-}\left[f_{opt1}^{+} - f_{opt1}^{-}\right] \quad (8b)$$

$$10 \cdot x_1^{-} - 12 \cdot x_2^{+} \leq 4{\cdot}2 - 0{\cdot}4 \cdot \lambda^{-} \quad (8c)$$

$$2{\cdot}8x_1^{-} + 3 \cdot x_2^{-} \leq 6{\cdot}5 - 0{\cdot}5 \cdot \lambda^{-} \quad (8d)$$

$$0 \leq \lambda^{-} \leq 1 \quad (8e)$$

Based on model (3), three violation variables are introduced into model (8) to relax its constraints. This results in the following formulation:

$$\text{Max } \lambda^{-} \quad (9a)$$

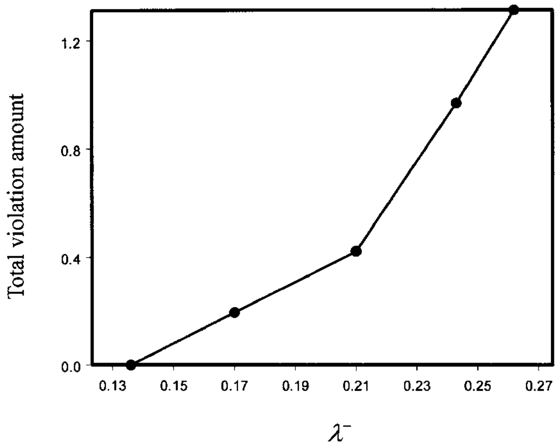

Figure 2. Total constraint-violation amounts under different λ^- levels.

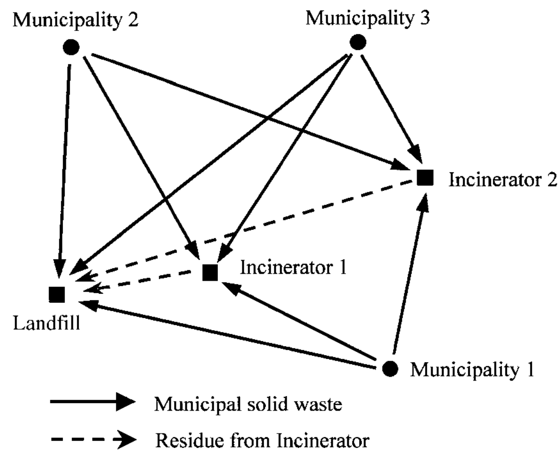

Figure 3. Overview of the study system.

Subject to

$$26 \cdot x_1^- - 6 \cdot x_2^+ + V_1 \geq f_{opt1}^- + \lambda^- \left[f_{opt1}^+ - f_{opt1}^- \right] \quad (9b)$$

$$10 \cdot x_1^- - 12 \cdot x_2^+ - V_2 \leq 4{\cdot}2 - 0{\cdot}4 \cdot \lambda^- \quad (9c)$$

$$2{\cdot}8 \cdot x_1^- + 3 \cdot x_2^+ - V_3 \leq 6{\cdot}5 - 0{\cdot}5 \cdot \lambda^- \quad (9d)$$

$$V_1 + V_2 + V_3 \leq \lambda^- \cdot TV \quad (9e)$$

$$\beta \leq \lambda^- \leq 1 \quad (9f)$$

where β denotes a minimum value of λ^-, with this value to be set by planners or decision makers. With varied TV and β levels, solutions with different λ^- values will be generated. Figure 2 shows the relationship between λ^- and TV. It is indicated that, with the increased TV level, the value of λ^- will also be raised. The results are useful for analyzing tradeoffs between the system-satisfaction levels and the associated risks of violating the fuzzy goal and/or constraints. If a decision maker desires to have a higher system-satisfaction level, they may have to face a higher risk of insufficiency in resource availability.

Application

The developed methodology was applied to the planning of waste flow allocation and facility expansion for a hypothetical but representative region. In the study system (as shown in Figure 3), there are three MSW management facilities available for three municipalities, including one landfill and two incinerators. The planning horizon is 15 years, which is further divided into three equal periods (5 years for each period). At the beginning of the planning horizon, the landfill has a capacity of $[0{\cdot}625, 0{\cdot}775] \times (10^6$ tonnes, while incinerators 1 and 2 have capacities of [100, 125] and [200, 225] tonne/day, respectively. Residues generated from the incinerators account for [20, 30]% of total incoming waste stream, and will be shipped to the landfill directly. Revenues from incinerators through energy recovery are in the range of [15, 25] dollars per tonne of waste incinerated. Table 1 gives waste generation rates in the three municipalities, Table 2 presents transportation costs for different waste delivery routes, and Table 3 gives operating costs of waste management facilities.

Table 1. Waste generation rate in each municipality

	Time Period		
	$k=1$	$k=2$	$k=3$
WG_{1k}	[200, 250]	[225, 275]	[250, 300]
WG_{2k}	[350, 400]	[375, 425]	[400, 450]
WG_{3k}	[275, 325]	[300, 350]	[325, 375]

When the capacities of the region's waste management facilities are insufficient for handling the incoming waste flows, system expansion becomes necessary. According to the region's environmental policy, the landfill can only be expanded once with an increment of $[1{\cdot}55, 2{\cdot}50] \times 10^6$ tonnes during the planning horizon. In comparison, each of the incinerators can be expanded once per period. Table 4 shows expansion options for the incinerators in different periods and the relevant costs for the expansions in present values.

The main problem under consideration is how to select preferred capacity expansion schemes for

Table 2. Waste transportation costs

	Time Period		
	$k=1$	$k=2$	$k=3$
Cost of waste transportation to the landfill ($/tonne)			
C_{11k} (municipality 1)	[12·1, 16·1]	[13·3, 17·7]	[14·6, 19·5]
C_{21k} (municipality 2)	[10·5, 14·0]	[11·6, 15·4]	[12·8, 16·9]
C_{31k} (municipality 3)	[12·7, 17·0]	[14·0, 18·7]	[15·4, 20·6]
Cost of waste transportation to incinerator 1 ($/tonne)			
C_{12k} (municipality 1)	[19·6, 12·8]	[10·6, 14·1]	[11·7, 15·5]
C_{22k} (municipality 2)	[10·1, 13·4]	[11·1, 14·7]	[12·2, 16·2]
C_{32k} (municipality 3)	[8·8, 11·7]	[9·7, 12·8]	[10·6, 14·0]
Cost of waste transportation to incinerator 2 ($/tonne)			
C_{13k} (municipality 1)	[12·1, 16·1]	[13·3, 17·7]	[14·6, 19·5]
C_{23k} (municipality 2)	[12·8, 17·2]	[14·1, 18·8]	[15·5, 20·7]
C_{33k} (municipality 3)	[4·2, 5·6]	[4·6, 6·2]	[5·1, 6·8]
Cost of residue transportation to the landfill ($/tonne)			
d_{2k} (incinerator 1)	[4·7, 6·3]	[5·2, 6·9]	[5·7, 7·6]
d_{3k} (incinerator 2)	[13·4, 17·9]	[14·7, 19·7]	[16·2, 21·7]
Revenue from incinerators ($/tonne)			
RE_k	[15, 25]	[15, 25]	[15, 25]

waste management facilities in different periods, and to allocate waste flows under different waste generation and management conditions, in order to minimize net system cost. Provision of several decision alternatives is desired since it will allow decision makers to examine tradeoffs between system satisfaction (with low net cost) and reliability (with low risk).

Information for many components in waste management systems is not known with certainty. Based on the formulation provided in model (2), the above problem can be formulated as an interval-fuzzy mixed integer linear programming (IFMILP) model as follows:

$$\text{Max } \lambda^{\pm} \tag{10a}$$

Subject to

$$1825 \cdot \sum_{i=1}^{3}\sum_{k=1}^{3}\left[\left(c_{i1k}^{\pm} + TE_{1k}^{\pm}\right)\cdot x_{i1k}^{\pm} + \sum_{j=2}^{3}\left(\begin{array}{l}\left(c_{ijk}^{\pm} + TE_{jk}^{\pm}\right)\cdot x_{ijk}^{\pm}\\ -RE_k^{\pm}\cdot x_{ijk}^{\pm}\\ +RF^{\pm}\cdot\left(d_{jk}^{\pm} + TE_{1k}^{\pm}\right)\cdot x_{ijk}^{\pm}\end{array}\right)\right] + \sum_{j=2}^{3}\sum_{m=1}^{3}\sum_{k=1}^{3}\left(FTC_{jmk}^{\pm}\cdot Z_{jmk}^{\pm}\right) + \sum_{k=1}^{3}\left(FLC_k^{\pm}\cdot Y_k^{\pm}\right) \le f_{opt1}^{+} - \lambda^{\pm}\cdot\left(f_{opt1}^{+} - f_{opt1}^{-}\right) \tag{10b}$$

(system cost constraint)

Table 3. Facility operating costs

	Time Period		
	$k=1$	$k=2$	$k=3$
TE_{1k} (Landfill)	[30, 45]	[40, 60]	[50, 80]
TE_{2k} (Incinerator 1)	[55, 75]	[60, 85]	[65, 95]
TE_{3k} (Incinerator 2)	[50, 70]	[60, 80]	[65, 85]

$$1825 \cdot \sum_{i=1}^{3}\sum_{k=1}^{k'}\left(x_{i1k}^{\pm} + RF^{\pm}\cdot\sum_{j=2}^{3}x_{ijk}^{\pm}\right) \le TL^{+} - \lambda^{\pm}\cdot\left(TL^{+} - TL^{-}\right) + \Delta TC\cdot\sum_{k=1}^{k'}Y_k^{\pm} \quad k' = 1, 2, 3 \tag{10c}$$

(landfill capacity constraints)

$$\sum_{i=1}^{3}x_{ijk'}^{\pm} \le WTE_j^{\,+} - \lambda^{\pm}\cdot\left(WTE_j^{+} - WTE_j^{-}\right) + \sum_{m=1}^{3}\sum_{k=1}^{k'}\left(Z_{jmk}^{\pm}\cdot\Delta TC_m\right) \quad j = 2, 3;\; k' = 1, 2, 3 \tag{10d}$$

(incinerator capacity constraints)

$$\sum_{j=1}^{3}x_{ijk}^{\pm} \ge WG_{ik}^{+} - \lambda^{\pm}\cdot\left(WG_{ik}^{+} - WG_{ik}^{-}\right) \quad i = 1, 2, 3;\; k = 1, 2, 3 \tag{10e}$$

(waste disposal constraints)

Table 4. Capacity expansion options for MSW management facilities

	Time Period		
	$k=1$	$k=2$	$k=3$
Capacity expansion option for incinerator j, $j=2,3$ (tonne/day)			
ΔTC_1 (option 1)	100	100	100
ΔTC_2 (option 2)	150	150	150
ΔTC_3 (option 3)	200	200	200
Capacity expansion option for the landfill (10^6 ton)			
ΔTC	[1·55, 2·5]	[1·55, 2·5]	[1·55, 2·5]
Capital cost for incinerator expansion, $j=2,3$ (\$$10^6$ present value)			
FTC_{j1k} (option 1)	10·5	8·3	6·5
FTC_{j2k} (option 2)	15·2	11·9	9·3
FTC_{j3k} (option 3)	19·8	15·5	12·2
Capital cost for landfill expansion (\$$10^6$ present value)			
FLC_k	[13, 15]	[13, 15]	[13, 15]

$$Y_k^{\pm} \le 1, \quad \ge 0, \quad = \text{integer}, \quad k = 1,2,3 \tag{10f}$$

(binary constraints on landfill expansion)

$$Z_{jmk}^{\pm} \le 1, \quad \ge 0, \quad = \text{integer}, \quad j = 2,3; \; k = 1,2,3; \; m = 1,2,3 \tag{10g}$$

(binary constraints on incinerator expansion)

$$\sum_{k=1}^{3} Y_k^{\pm} \le 1 \tag{10h}$$

(landfill capacity can expand only once during the planning horizon)

$$\sum_{m=1}^{3} Z_{jmk}^{\pm} \le 1 \quad j = 2,3; \; k = 1,2,3 \tag{10i}$$

(capacity of each incinerator can expand only once during each planning period)

$$x_{ijk}^{\pm} \ge 0 \quad \forall i,j,k,l \tag{10j}$$

(technical constraints)

where:

i = index for municipalities, $i = 1, 2, 3$;
j = index for MSW management facilities, $j = 1, 2, 3$;
k = index for periods, $k = 1, 2, 3$;
m = index for facility capacity expansion type, $m = 1, 2, 3$;
$RF^{\pm}$ = residue flow-rate from incinerators to landfill;
$x_{ijk}^{\pm}$ = solid waste stream from municipal i to facility j in period k (tonne/d);
$\Delta TC^{\pm}$ = total amount of expansion capacity for landfill (tonne);
$Y_k^{\pm}$ = integer variable for landfill expansion in period k;
$TL^{\pm}$ = existing capacity of landfill (tonne);
$WTE_j^{\pm}$ = existing capacity of incinerator j ($j=2$ and 3) (tonne/day);
$Z_{jmk}^{\pm}$ = integer variable for expansion of incinerator j in period k;
ΔTC_m = amount of each expansion type for incinerators (tonne/day);
$c_{ijk}^{\pm}$ = waste transportation costs from municipality i to facility j in period k (\$/tonne);
$d_{jk}^{\pm}$ = residue transportation costs from incinerator j to landfill in period k (\$/tonne);
$TE_{jk}^{\pm}$ = operating costs of MSW management facility j in period k (\$/tonne);
$RE_k^{\pm}$ = revenue of unit MSW incinerated in period k (\$/tonne);
$FLC_k^{\pm}$ = unit capacity expansion cost for landfill in period k (\$/tonne);
$FTC_{jmk}^{\pm}$ = unit cost of capacity expansion type m for incinerator j in period k (\$/tonne);
$WG_{ik}^{\pm}$ = daily generation amount of solid waste in municipal i in period k (tonne/day);

A number of control parameters (for creating a fuzzy goal and constraints) can be introduced to model (10) for obtaining alternative solutions under various levels of constraint violation. Higher $\lambda^{\pm}$ values correspond to less strict model constraints, which can be interpreted as more optimistic conditions with lower system reliabilities; at the same time, less strict model constraints will result in lower system net costs. Thus, an analysis of tradeoffs between the objective-function value (i.e. net system cost) and the system reliability can be undertaken. A constraint-relaxed IFMILP model can then be formulated as follows:

$$\text{Max } \lambda^- \tag{11a}$$

Subject to

$$\frac{2V_1}{(f_{opt1}^+ + f_{opt1}^-)} + \sum_{i=2}^{4} \frac{2V_i}{(TL^+ + TL^-)} + \sum_{i=5}^{7} \frac{2V_i}{(WTE_2^+ + WTE_2^-)} + \sum_{i=8}^{10} \frac{2V_i}{(WTE_3^+ + WTE_3^-)} + \sum_{j=1}^{3} \left(\sum_{i=11+3(j-1)}^{13+3(j-1)} \frac{2V_i}{(WG_j^+ + WG_j^-)} \right) \leq \lambda^- \cdot TV \tag{11b}$$

(total violation constraint)

$$1825 \cdot \sum_{i=1}^{3} \sum_{k=1}^{3} \left[\left(c_{i1k}^+ + TE_{1k}^+\right) \cdot x_{i1k}^+ + \sum_{j=2}^{3} \begin{pmatrix} \left(c_{ijk}^+ + TE_{jk}^+\right) \cdot x_{ijk}^+ \\ -RE_k^- \cdot x_{ijk}^+ \\ +RF^+ \cdot \left(d_{jk}^+ + TE_{1k}^+\right) \cdot x_{ijk}^+ \end{pmatrix} \right] + \sum_{j=2}^{3} \sum_{m=1}^{3} \sum_{k=1}^{3} \left(FTC_{jmk}^+ \cdot Z_{jmk}^+\right) + \sum_{k=1}^{3} (FLC_k^+ \cdot Y_k^+) - V_1 \leq f_{opt1}^+ - \lambda^- \cdot \left(f_{opt1}^+ - f_{opt1}^-\right) \tag{11c}$$

(relaxed constraint on system cost)

$$1825 \cdot \sum_{i=1}^{3} \sum_{k=1}^{k'} \left(x_{i1k}^+ + RF^+ \cdot \sum_{j=2}^{3} x_{ijk}^+ \right) - V_{k'+1} \leq TL^+ - \lambda^- \cdot (TL^+ - TL^-) + \Delta TC \cdot \sum_{k=1}^{k'} Y_k^+ \quad k' = 1, 2, 3 \tag{11d}$$

(relaxed constraints on landfill capacities)

$$\sum_{i=1}^{3} x_{ijk'}^+ - V_{k'+3(j-1)+1} \leq WTE_j^+ - \lambda^- \left(WTE_j^+ - WTE_j^-\right) + \sum_{m=1}^{3} \sum_{k=1}^{k'} (Z_{jmk}^+ \cdot \Delta TC_m) \quad j = 2, 3; \; k' = 1, 2, 3 \tag{11e}$$

(relaxed constraints on incinerator capacities)

$$\sum_{j=1}^{3} x_{ijk}^+ + V_{k+3(i-1)+10} \geq WG_{ik}^+ - \lambda^- \cdot \left(WG_{ik}^+ - WG_{ik}^-\right) \quad i = 1, 2, 3; \; k = 1, 2, 3 \tag{11f}$$

(relaxed constraints on waste generation and disposal)

$$Y_k^+ \leq 1, \quad \geq 0, \quad = \text{integer}, \quad k = 1, 2, 3 \tag{11g}$$

(binary constraints on landfill expansion)

$$Z_{jmk}^+ \leq 1, \quad \geq 0, \quad = \text{integer}, \quad j = 2, 3; \; k = 1, 2, 3; \; m = 1, 2, 3 \tag{11h}$$

(binary constraints on incinerator expansion)

$$\sum_{k=1}^{3} Y_k^+ \leq 1 \tag{11i}$$

(landfill capacity can expand only once during the planning horizon)

$$\sum_{m=1}^{3} Z_{jmk}^+ \leq 1 \quad j = 2, 3; \; k = 1, 2, 3 \tag{11j}$$

(capacity of each incinerator can expand only once during each planning period)

$$x_{ijk}^+ \geq x_{ijk_{opt}}^- \qquad \forall i, j, k \tag{11k}$$

$$x_{ijk}^+ \geq 0, \qquad V_l \geq 0 \;\forall i, j, k, l \tag{11l}$$

(technical constraints)

where V_l denotes violation level for constraint l, $l = 1, 2, \ldots, 19$; and $x_{ijk_{opt}}^-$ is the waste flow allocation from the sub-model corresponding to λ^+ (Huang *et al.*, 1995). Here, due to different units of violation variables, they are normalized by dividing them by the mean values of the interval of the right-hand side constraints and objective function.

Result Analysis

Solution of IFMILP model (10)

Solutions from the IFMILP model (10) regarding waste flow allocation are presented in Table 5. It is indicated that, in period 1, wastes from municipality

Table 5. Solution of IFMILP model

Municipality	Facility	Period	Symbol	Solution
Municipality 1	Flow to landfill (tonne/day)	1	$x_{111}^{\pm}$	[77, 119]
		2	$x_{112}^{\pm}$	[0, 42]
		3	$x_{113}^{\pm}$	[0, 42]
Municipality 2	Flow to landfill (tonne/day)	1	$x_{211}^{\pm}$	[256, 298]
		2	$x_{212}^{\pm}$	[203, 245]
		3	$x_{213}^{\pm}$	[278, 320]
Municipality 3	Flow to landfill (tonne/day)	1	$x_{311}^{\pm}$	0
		2	$x_{312}^{\pm}$	0
		3	$x_{313}^{\pm}$	0
Municipality 1	Flow to incinerator 1 (tonne/day)	1	$x_{121}^{\pm}$	0
		2	$x_{122}^{\pm}$	227
		3	$x_{123}^{\pm}$	177
Municipality 2	Flow to incinerator 1 (tone/day)	1	$x_{221}^{\pm}$	96
		2	$x_{222}^{\pm}$	74
		3	$x_{223}^{\pm}$	124
Municipality 3	Flow to incinerator 1 (tonne/day)	1	$x_{321}^{\pm}$	0
		2	$x_{322}^{\pm}$	0
		3	$x_{323}^{\pm}$	0
Municipality 1	Flow to incinerator 2 (tonne/day)	1	$x_{131}^{\pm}$	125
		2	$x_{132}^{\pm}$	0
		3	$x_{133}^{\pm}$	75
Manicipality 2	Flow to incinerator 2 (tonne/day)	1	$x_{231}^{\pm}$	0
		2	$x_{232}^{\pm}$	100
		3	$x_{233}^{\pm}$	0
Municipality 3	Flow to incinerator 2 (tonne/day)	1	$x_{331}^{\pm}$	277
		2	$x_{332}^{\pm}$	302
		3	$x_{333}^{\pm}$	327
$\lambda^{\pm}$			[0·124, 0·959]	
System cost ($\$10^6$)			[306·204, 552·628]	

1 should be shipped to either the landfill or incinerator 2; those from municipality 2 should be mainly shipped to the landfill; in comparison, all wastes from municipality 3 should be transported to incinerator 2 due to its vicinity to this facility.

In period 2, the planned waste flow allocation pattern is different from the last period. The majority of wastes from municipality 1 would be shipped to incinerator 1; those from municipality 2 should have over 50% shipped to the landfill, with the remaining being delivered to either incinerator 1 or 2; all wastes from municipality 3 should still be transported to incinerator 2.

In period 3, the majority of wastes from municipality 1 would be shipped to incinerator 1; those from municipality 2 should be shipped to the landfill and incinerator 1; and all wastes from municipality 3 should still be transported to incinerator 2.

The solutions for integer variables are provided in Table 6 (only those that are not equal to zero are listed). It is indicated that, in period 1, the landfill should be expanded with an increment of [1·55, 2·50] million tonnes; incinerator 1 should have a capacity expansion of 200 tonne/day in period 2; and incinerator 2 should be expanded with an incremental capacity of 150 tonne/day in period 1. The net system cost is $[306·2, 552·6] million, with the $\lambda^{\pm}$ range being [0·124, 0.959]. The lower bound of $\lambda^{\pm}$ is only 0·124, which indicates a relatively low possibility of satisfying the objective function and constraints.

Table 6. Solution of integer variables

Facility	Period	Capacity increment	Variable	Solution
Landfill	1	$[1{\cdot}55, 2{\cdot}50] \times 10^6$ tonne/day	Y_1	1
Incinerator 1	2	200 tonne/day	Z_{232}	1
Incinerator 2	1	150 tonne/day	Z_{321}	1

Solution of constraint-relaxed IFMILP model (11)

Through solving model (11) under various levels of allowable violations for the constraints, relationships between λ^{-} levels and the constraint-violation

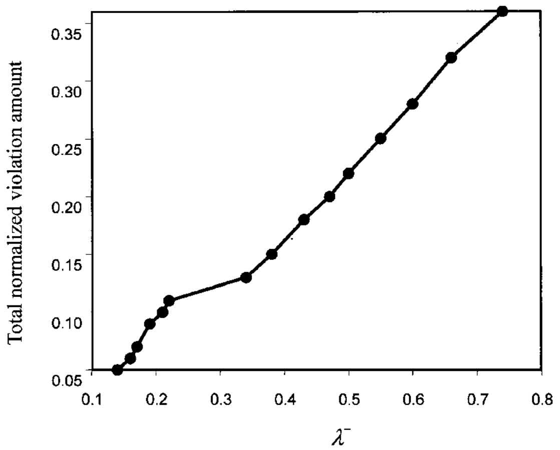

Figure 4. Relationship between total normalized constraint-violation amount and λ^- level.

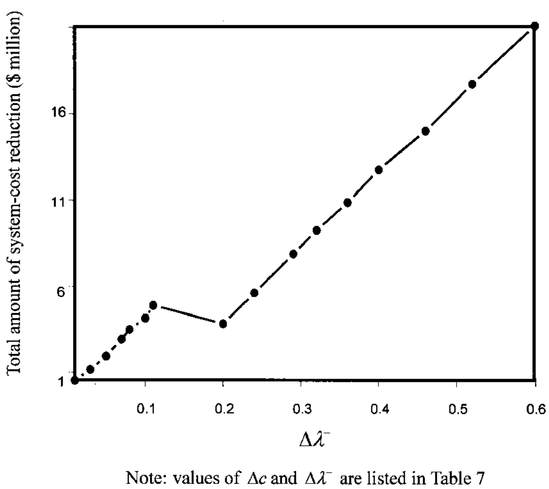

Figure 6. Reduced system cost under different $\Delta\lambda^-$ levels.

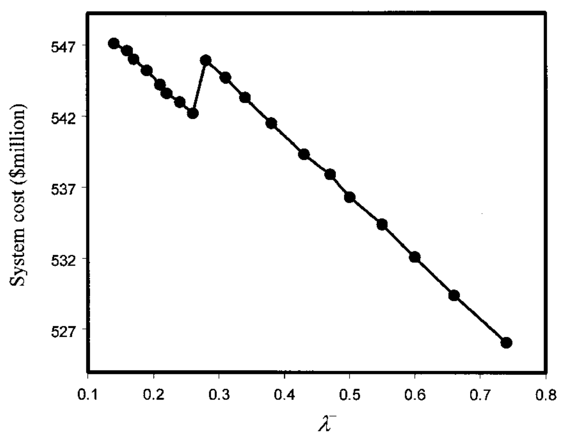

Figure 5. Variations of system cost under different λ^- levels.

levels can be obtained, as presented in Figure 4. A higher λ^- level would lead to a higher total normalized violation level.

Solution of model (11) can also help to quantify the required violation amount for each constraint, given a reduced λ^- level. This would provide useful information regarding relationships between system costs and violation levels, as shown in Figure 5. In general, the system cost decreases as the λ^- level (or total violation level) increases. With the λ^- increasing, the coefficients of the right-hand constraints of the capacities of treatment facilities decrease, which results in different types of capacity expansion of treatment facilities. Here, for incinerator 2, the capacity expansion will change from type 2 to type 3 at time period 1. That is, the capacity expansion will be 200 tonne/day, not 150 tonne/day, when λ^- is greater than a certain value. Therefore, the system cost was increased and the jump occurred as depicted in the Figure 5. This is the same reason for the drops or jumps appearing in other figures and tables.

Through solving model (11), system-cost under different λ^- levels can be obtained as shown in Table 7. A total of 19 scenarios corresponding to 19 λ^- levels were analyzed to obtain insight into the variations of system-cost reduction under different λ^- levels. In the analysis, the condition of $\lambda_0^- = 0{\cdot}14$ is used as the reference scenario [rather than using the lower bound of λ $(=0{\cdot}124)$], since there is no constraint violation under that λ^- level. Table 7 also lists the ratio of system-cost reduction (Δc_i) to λ^- value variation ($\Delta\lambda_i^-$) under each scenario (where $\Delta\lambda_i^- = \lambda_i^- - \lambda_0^-$, $\Delta c_i = c_0 - c_i$, $i = 1, 2, \ldots, 18$). The relationships between Δc_i and $\Delta\lambda_i^-$ are also depicted in Figure 6. The results indicate that the values of $\Delta c_i/\Delta\lambda_i^-$ do not vary significantly when λ^- is low; however, the $\Delta c_i/\Delta\lambda_i^-$ drops dramatically when $\lambda^- > 0{\cdot}258$, and then keeps increasing in the interval of 0·280 to 0·742. It implies that, if decision-makers have to make compromises between economic and environmental objectives, there would be more chances for reducing system costs under high λ^- levels. Nevertheless, the high λ^- level corresponds to a higher risk of violating system constraints (including environmental constraints).

The λ^- levels correspond to decision makers' preferences regarding environmental and economic tradeoffs. Decisions at lower λ^- levels would lead to increased reliability in fulfilling system requirements, but with a higher cost; in comparison,

Table 7. Analysis of system-cost reduction

Selected levels of λ^-	λ_0^-	λ_1^-	λ_2^-	λ_3^-	λ_4^-	λ_5^-	λ_6^-	λ_7^-	λ_8^-	λ_9^-
	0·143	0·156	0·171	0·189	0·211	0·225	0·240	0·258	0·280	0·307
System-cost (c) ($ million)	c_0	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9
	547·1	546·6	546·0	545·2	544·2	543·6	543·0	542·2	545·9	544·7
$\Delta\lambda^-$		$\Delta\lambda_1^-$	$\Delta\lambda_2^-$	$\Delta\lambda_3^-$	$\Delta\lambda_4^-$	$\Delta\lambda_5^-$	$\Delta\lambda_6^-$	$\Delta\lambda_7^-$	$\Delta\lambda_8^-$	$\Delta\lambda_9^-$
		0·013	0·028	0·046	0·068	0·082	0·097	0·115	0·137	0·164
System-cost reduction (Δc) ($ million)		Δc_1	Δc_2	Δc_3	Δc_4	Δc_5	Δc_6	Δc_7	Δc_8	Δc_9
		0·5	1·1	1·9	2·9	3·5	4·1	4·9	1·2	2·4
$\Delta c/\Delta\lambda^-$ ($ million per unit change of λ)		38·5	39·3	41·3	42·6	42·7	42·3	42·6	8·8	14·6

Selected levels of λ^-	λ_{10}^-	λ_{11}^-	λ_{12}^-	λ_{13}^-	λ_{14}^-	λ_{15}^-	λ_{16}^-	λ_{17}^-	λ_{18}^-
	0·340	0·381	0·434	0·467	0·504	0·548	0·600	0·664	0·742
System-cost (c) ($ million)	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}
	543·3	541·5	539·3	537·9	536·3	534·4	532·1	529·4	526·1
$\Delta\lambda^-$	$\Delta\lambda_{10}^-$	$\Delta\lambda_{11}^-$	$\Delta\lambda_{12}^-$	$\Delta\lambda_{13}^-$	$\Delta\lambda_{14}^-$	$\Delta\lambda_{15}^-$	$\Delta\lambda_{16}^-$	$\Delta\lambda_{17}^-$	$\Delta\lambda_{18}^-$
	0·197	0·238	0·291	0·324	0·361	0·405	0·457	0·521	0·599
System-cost reduction (Δc) ($ million)	Δc_{10}	Δc_{11}	Δc_{12}	Δc_{13}	Δc_{14}	Δc_{15}	Δc_{16}	Δc_{17}	Δc_{18}
	3·8	5·6	7·8	9·2	10·8	12·7	15·0	17·7	21·0
$\Delta c/\Delta\lambda^-$ ($ million per unit change of λ)	19·3	23·5	26·8	28·4	29·9	31·4	32·8	34·0	35·0

Note: $\Delta\lambda_i^- = \lambda_i^- - \lambda_0^-$, and $\Delta c_i = c_i - c_0$, where $i = 1, 2, \ldots, 18$.

decisions at higher λ^- levels would result in a lower system cost, but the risk of violating the system constraints would be increased.

These characteristics are demonstrated in the summary of the modeling solutions as shown in Table 8. Changes in system costs under different violation levels with regard to the capacity of incinerator 2 (i.e. different λ^- values) are provided. When decision-makers prefer a satisfaction level of $\lambda^- = 0{\cdot}24$, the corresponding system cost will be \$543·0 million, with 14·1 tonne/day of waste flow exceeding the capacity of incinerator 2. In comparison, when the satisfaction level is reduced to $\lambda^- = 0{\cdot}14$, the corresponding system cost will be increased to \$547·1 million, with the violation amount (for incinerator 2) being reduced to 9·2 tonne/day. In Table 8, the violation to incinerator 2 becomes zero after $\lambda^- = 0{\cdot}26$. The reason is that the larger expansion capacity was implemented at incinerator 2 at this point, and the total amount of generated solid waste decreases with λ^- increasing; therefore, the violation to incinerator 2 drops to zero.

We can define c^+ as the tolerable limit for system cost corresponding to λ^-. Four scenarios corresponding to c^+, 98% of c^+, 96% of c^+, and 94% of c^+ were analyzed. The results are provided in Table 9 and Figure 7, showing the relationships between system costs and λ^- values (and thus constraint-violation levels). From Table 9, we can see that a lower c^+ limit can result in a higher λ^- level with no system violation. Figure 7 indicates that the differences between adjacent scenarios are greater than 2%. Therefore, system costs can be decreased through improving cost-effectiveness and efficiencies of management facilities. The drops and jumps in Figure 7 and Table 9 are caused by the integer variables as discussed previously.

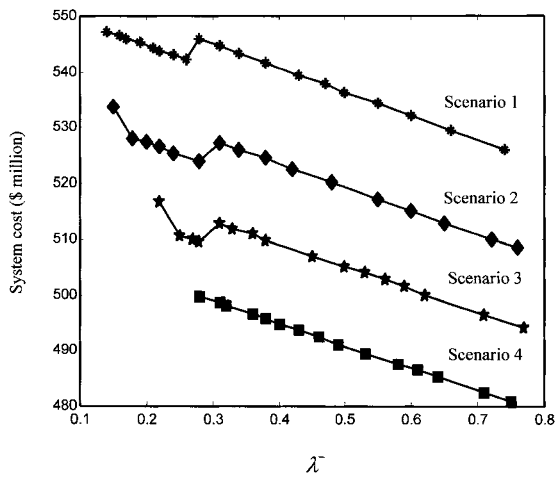

Figure 7. System costs under different λ^- levels.

Table 8. System costs and constraint-violation amounts under different λ^- levels

	λ^- level													
	0·14	0·17	0·19	0·21	0·22	0·24	0·26	0·34	0·38	0·47	0·55	0·60	0·66	0·74
Objective function value ($\$10^6$)	547·1	546·0	545·2	544·2	543·6	543·0	542·2	543·3	541·5	537·9	534·4	532·1	529·4	526·1
Level of violating incinerator 2's capacity (consistent for all three periods) (tonne/day)	9·2	10·6	11·5	12·6	13·3	14·1	15·0	0	0	0	0	0	0	0

Table 9. System costs and total constraint-violation levels under different λ^- values

Scenario 1								
System cost ($ million)	547·2	543·6	542·2	543·3	537·9	534·4	529·4	526·1
λ^-	0·14	0·22	0·26	0·34	0·47	0·55	0·66	0·74
Constraint-violation level (tonne/day)	27·7	39·9	44·8	0	0	0	0	0
Scenario 2								
System cost ($ million)	533·7	527·4	525·4	526·0	520·2	517·1	512·8	508·5
λ^-	0·15	0·20	0·24	0·34	0·48	0·55	0·65	0·76
Constraint-violation level (tonne/day)	0	29·7	36·8	0	0	0	0	0
Scenario 3								
System cost ($ million)	516·6	510·0	509·5	507·0	505·1	501·5	500·0	494·1
λ^-	0·22	0·27	0·28	0·45	0·50	0·59	0·62	0·77
Constraint-violation level (tonne/day)	0	40·2	42·3	0	0	0	0	0
Scenario 4								
System cost ($ million)	499·7	498·0	496·6	492·6	489·5	487·6	485·3	480·9
λ^-	0·28	0·32	0·36	0·46	0·53	0·58	0·64	0·75
Constraint-violation level (tonne/day)	0	0	0	0	0	0	0	0

Regret assessment

'Regret' is defined as the difference between the system cost obtained from the model under a given λ^- level and the system cost which would have been spent if the actual scenario had happened; or the difference between total violation to system constraints under a given λ^- level and the total violation that would have been produced if the actual scenario had happened. Regret analysis will enable the determination of the relative magnitudes of excessive or insufficient waste disposal capacities that would result from choosing one certain level of λ^- that proves to be inaccurate or inapplicable.

Table 10 gives regret assessment results under 9 selected λ^- levels. Columns 1 to 4 list system cost, average amount of solid waste generation, and violation to constraints of facility capacities under 9 selected λ^- levels; at the same time, rows 1 to 4 list system cost, average amount of solid waste generation, and violation to constraints of facility capacities under the same 9 λ^- levels. This allows pairwise comparisons of results under different λ^- levels. Thus, given a selected λ^- level, the corresponding regrets (i.e. excess system costs and amounts of solid waste untreated) under each decision scenario with a realized λ^- level can be identified. For example, examine a λ^- level of 0·22, the regret will be zero when the decision scheme is associated with a realized λ^- level of 0·22. However, if the realized λ^- level is 0·43, the regret of excess system cost for selecting 0·22 will become \$4·37 million (i.e. system cost at the λ^- level of 0·22 − system cost at the λ^- level of 0·43 = \$543·64 million − \$539·27 million = \$4·37 million); there is no regret of solid waste untreated, because the waste treatment capacity at the given $\lambda^- = 0{\cdot}22$ is higher than that at the realized $\lambda^- = 0{\cdot}43$. Reversely, examine a λ^- level of 0·43, the regret of excess system cost is zero under a realized λ^- level of 0·22, but the regret of solid waste untreated is 44·81 tonne/day (i.e. average amount of solid waste generation + violation at the λ^- level of 0·22 − average amount of solid waste generation − violation at the λ^- level of 0·43 = 1017·0 + 13·31 − 985.5 − 0 = 44·81 tonne/day).

Table 10. Results of regret assessment

Examined λ^{-} level	Examined system cost ($ million)	Examined average amount of solid waste generation (tonne/day)	Examined violation to constraints of facility capacities (tonne/day)	Realized λ^{-} level:									Aggregated Results[d]	
				0·14	0·19	0·22	0·26	0·31	0·43	0·55	0·60	0·74	Excess system cost ($ million)	Amount of solid waste untreated (tonne/day)
				Realized system cost ($million):										
				547·15	545·18	543·64	542·24	544·73	539·27	534·39	532·15	526·06		
				Realized average amount of solid waste generation (tonne/day):										
				1029·0	1021·5	1017·0	1011·0	1003·5	985·5	967·5	960·0	939·0		
				Realized violation to constraints of facility capacities (tonne/day):										
				9·22	11·51	13·31	14·95	0	0	0	0	0		
0·14	547·15	1029·0	9·22		+1.97[a]	+3·51	+4·91	+2·42	+7·88	+12·76	+15·00	+21·09	+8·69	0
0·19	545·18	1021·5	11·51	−5·21[b]		+1·54	+2·94	+0·45	+5·91	+10·79	+13·03	+19·12	+6·72	−0·65
0·22	543·64	1017·0	13·31	−7·91	−2·70		+1·40	0[c]	+4·37	+9·25	+11·49	+17·58	+5·51	−1·33
0·26	542·24	1011·0	14·95	−12·27	−7·06	−4·36		0	+2·97	+7·85	10·09	16·18	+4·64	−2·96
0·31	544·73	1003·5	0	−34·72	−29·51	−26·81	−22·45		+5·46	+10·34	+12·58	+18·67	+5·88	−14·19
0·43	539·27	985·5	0	−52·72	−47·81	−44·51	−40·45	−18·00		+4·88	+7·12	+13·21	+3·15	−25·44
0·55	534·39	967·5	0	−70·72	−65·51	−62·81	−58·45	−36·00	−17·00		+2·24	+8·33	+1·32	−38·94
0·60	532·15	960·0	0	−78·22	−73·01	−70·31	−66·95	−43·50	−25·50	−7·50		+6·09	+0·76	−45·50
0·74	526·06	939·0	0	−99·22	−94·01	−91·31	−86·95	−64·50	−46·50	−28·50	−21·00		0	66·50

[a] The positive signs mean excess system costs ($ million).
[b] The negative signs mean amounts of solid waste untreated (tonne/day).
[c] The drops occurred here due to the same reason discussed previously in the text; there is no regret so that zero is used.
[d] The aggregated results are average values of the 8 examined λ^{-} levels.

The aggregated regret values regarding excess system cost or solid waste untreated under different λ^- levels are also provided in Table 10. For each λ^- level, its aggregated regret value is equal to the average of the other 8 λ^- levels. For example, the aggregated value of excess system cost is $(1{\cdot}97 + 3{\cdot}51 + 4{\cdot}91 + 2{\cdot}42 + 7{\cdot}88 + 12{\cdot}76 + 15{\cdot}00 + 21{\cdot}09)/8 = 8{\cdot}69$ when the scenario of $\lambda^- = 0{\cdot}14$ is selected (see Table 10). This analysis is based on an assumption that each scenario has the same probability of occurrence. Thus, the solutions can be generally sorted into three groups. The first group consists of λ^- levels ranging from 0·14 to 0·26, characterized by small amount of solid waste untreated and relatively large excess system cost. The second group is in the range of $\lambda^- = 0{\cdot}60$ to 0·74, which is characterized by negligible excess system cost with relatively large amount of solid waste untreated. The third group ranges from 0·31 to 0·55, which represents a compromise between system cost and reliability.

Thus, lower λ^- levels would result in higher system costs, lower constraint-violation levels, higher system reliability, and lower system risk. In comparison, higher λ^- levels sacrifice system reliability to obtain lower system costs. The modeling results can therefore provide useful information for making compromises among various objectives. Generally, solutions with λ^- values located within 0·31 to 0·55 (i.e. the third group) may be recommended. For alternatives in this group, the regret value for system cost is considered reasonable while the amount of solid waste untreated is moderate.

Conclusions

In this study, an interval-parameter violation analysis approach based on methods of IFMP and regret analysis is proposed for violation analyses of environmental systems under uncertainty. In this methodology, several given levels of tolerable violation for system constraints are permitted. This is realized through relaxing the critical constraints using violation variables, such that the model's decision space can be expanded. Thus, solutions from the violation analysis will not necessarily satisfy all of the model's original constraints.

The method was applied to the planning of a municipal solid waste management system. The results indicate that potentially useful information is obtained through this developed approach. Generally, lower λ^- levels would result in higher system costs, lower constraint-violation levels, higher system reliability, and lower system risk. The modeling results help to generate a number of decision alternatives under various system conditions, allowing more in-depth analyses of tradeoffs between environmental and economic objectives as well as those between system optimality and reliability. The general approach is applicable to a wide range of environmental problems where uncertainty exists for input parameters and considerable care is needed to balance off environmental and economic objectives.

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Appendix: Nomenclature

$A^\pm$	$A^\pm \in \{R^\pm\}^{m\times n}$, $R^\pm$ denotes a set of interval numbers
$B^\pm$	$B^\pm \in \{R^\pm\}^{m\times l}$, $R^\pm$ denotes a set of interval numbers
$c^\pm$	the tolerable limit for system cost corresponding to λ^-
$C^\pm$	$C^\pm \in \{R^\pm\}^{l\times n}$, $R^\pm$ denotes a set of interval numbers
${c_{jk}}^\pm$	waste transportation costs from municipality i to facility j in period k (\$/tonne)
${d_{jk}}^\pm$	residue transportation costs from incinerator j to landfill in period k (\$/tonne)
${FLC_k}^\pm$	unit capacity expansion cost for landfill in period k (\$/tonne)
f_{opt1}^-, f_{opt1}^+	the most and least desirable system objectives, respectively
$FTC_{jmk}^\pm$	unit cost of capacity expansion type m for incinerator j in period k (\$/tonne)
i	index for municipalities, $i = 1, 2, 3$
j	index for MSW management facilities, $j = 1, 2, 3$
k	index for periods, $k = 1, 2, 3$
m	index for facility capacity expansion type, $m = 1, 2, 3$
$RE_k^\pm$	revenue of unit MSW incinerated in period k (\$/tonne)
$RF^\pm$	residue flow-rate from incinerators to landfill
$TE_{jk}^\pm$	operating costs of MSW management facility j in period k (\$/tonne)
$TL^\pm$	existing capacity of landfill (tonne)

TV	the total tolerable violation limit
V_l	denotes violation level for constraint l, $l = 1, 2, \ldots, 19$
$WG_{ik}^{\pm}$	daily generation amount of solid waste in municipal i in period k (tonne/day)
$WTE_j^{\pm}$	existing capacity of incinerator j ($j = 2$ and 3) (tonne/day)
$X^{\pm}$	$X^{\pm} \in \{R^{\pm}\}^{n \times l}, R^{\pm}$ denotes a set of interval numbers
$x_{ijk}^{\pm}$	solid waste stream from municipal i to facility j in period k (tonne/d)
$x_{ijk_{opt}}^{-}$	the waste flow allocation from the submodel corresponding to λ^{+}
$Y_k^{\pm}$	integer variable for landfill expansion in period k
$Z_{jmk}^{\pm}$	integer variable for expansion of incinerator j in period k
$\Delta TC^{\pm}$	total amount of expansion capacity for landfill (tonne)
$\Delta TC^{\pm}$	ΔTC_m amount of each expansion type for incinerators (tonne/day)
β	denotes a minimum value of λ^{-}
$\lambda^{\pm}$	control variable corresponding to the degree (membership grade) to which the model's solution fulfills the fuzzy goal or constraints